ON CROSS-PROCESSING SCALAR AND VECTOR QUANTITIES IN UNDERWATER ACOUSTICS

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INTRODUCTION

Considerable studying is being given to acoustic underwater ambient noise last 50 years. However, with rare exception, they employed systems built on the base of scalar sensors of the sound pressure, i.e. hydrophones.

While investigating energetics and directivity of underwater ambient noise from sound pressure data acquired, the conclusions are being drawn from the potential energy density E_p properties. In this connection, the "sound field intensity" term used to be identified with the potential energy density. Since the sound field intensity is, by definition, a vector quantity, Refs 1, 2, and 3, a considerable amount of information on actual acoustic field, directly connected to the vector nature of the intensity, appears to be overlooked in scalar measurements.

The intensity $I = <|p(t)|^2>$ calculated from sound pressure measurements is referred to as a scalar intensity, in contrast to the vector intensity $\vec{I} = < p(t)\vec{V}(t) >$, where $p(t)$ and $\vec{V}(t)$ are instantaneous acoustic pressure and particle velocity in the medium respectively and <…> represents the time-averaging operation. Most modern underwater ambient noise investigations never consider the vector intensity unlike similar noise studies in modern technical aero-acoustics.

Scalar approach holds good in particular cases of doing measurements in single plane traveling wave field or in the far zone of a single traveling spherical wave in which sound pressure and particle velocity are related as $p = \pm\rho c V$, where ρ is the density of the medium and c is the sound speed.

There is an erroneous opinion that simultaneous measurements of the acoustic pressure and particle velocity vector at a given point in the ocean appear to be required but in the near field of sound sources. However, once two plane waves of the same frequency superimpose, the net particle velocity vector motion is no longer longitudinal in a general way. In a given case, the net particle velocity vector rotates describing elliptical trajectory, i.e. there are vortexes to be found in a given velocity field. Consequently, by analogy with electromagnetic field, the term "polarization" is also applicable for acoustic fields in fluids or gases to describe vortex moments of the medium particles in the acoustic wave. Elliptical polarization is a general case of polarization, so, just in the particular case of a plane wave, there exist linear polarization constant all over the sound field, i.e. no vortexes are to be found in the velocity field. This suggests the sound field description with the scalar potential $\Phi(x,y,z)$ is not always applicable. Refs 4, 5, and 6 call the reader's attention to the problem of interest and discuss the conditions imposed on the initial wave equation,

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \Delta\varphi = 0 \tag{1}$$

while modifying it to Helmholts equation.

$$\varphi(x,y,z,t) = \Phi(x,y,z)\cos\omega t + \Psi(x,y,z)\sin\omega t \tag{2}$$

Insert Eq. 2 into Eq. 1

Both $\Phi(x,y,z)$ and $\Psi(x,y,z)$ have two solutions.

$$\Delta\Phi + k^2\Phi = 0, \qquad \Delta\Psi + k^2\Psi = 0, \tag{3}$$

this points to elliptical trajectory of the medium particles moving in the acoustic field.

$$\varphi = A(x,y,z)\cos\omega(t - \tau(x,y,z)) \tag{4}$$

One can get linear trajectory of the medium particles by inserting Eq. 4 into Eq. 1. The Eq. 4 sets a condition on the solution of Eq. 3 for unidirectional $\nabla\Phi$ and $\nabla\Psi$ vectors. In Ref. 6 such waves are referred to as "simple" waves. The acoustic fields we are dealing with in experimental underwater acoustics, are by no means "simple", that should be taken into account while doing studying.

Generally, the particle velocity vector $\vec{V}$ can be given as, see Ref. 4,

$$\vec{V} = \text{-grad}\Phi + \text{rot}\,\Psi, \tag{5}$$

where Φ and Ψ are scalar and vector velocity potentials.

While studying acoustic fields in actual mediums, one needs a complete set of basic quantities available, the potential energy density E_p, the kinetic energy density E_κ, the intensity vector $\vec{I}$, i.e. energy flux density vector or Umov vector, and the impulse flux density vector. These quantities are united in the impulse-energy tensor of moving fluid, Ref. 2.

According to above-stated, we will name the field of research as "scalar acoustics" while dealing with the scalar intensity or we will name the field of research as "vector acoustics" while doing research employing the vector intensity.

Vector acoustics is based on simultaneous measurements of four physical quantities at a given point in the sound field. The quantities are, the acoustic pressure, *p(t)* and three orthogonal components of the particle velocity vector $\vec{V}(t)\{V_x(t), V_y(t), V_z(t)\}$ in the acoustic wave.

Acoustic sensor incorporating omnidirectional sensor of the acoustic pressure *p(t)*, i.e. scalar sensor, along with 3-component particle velocity sensor capable of measuring three orthogonal components of the particle velocity vector $\vec{V}(t)\{V_x(t), V_y(t), V_z(t)\}$, i.e. vector sensor, is referred to as combined sensor, Ref. 11.

1. CORRELATION RELATIONSHIPS

While describing the acoustic pressure field $p(t,\vec{r})$ within the framework of scalar approach as a zero-meanth centered random process one can use the temporal-spatial correlation function

$$R_{p^2}(t_1,t_2,\vec{r}_1,\vec{r}_2) = \langle\{p(t_1,\vec{r}_1)p(t_2,\vec{r}_2\}\rangle, \tag{6}$$

where $\vec{r}_1$ and $\vec{r}_2$ denote two measurement points, t_1 and t_2 are two instants of measurements, <…> presents the spatial and temporal averaging operation.

The choice of thermodynamic quantity, the acoustic pressure $p(t,\ \vec{r})$ for description the acoustic field is motivated by the fact that the pressure is the most useful and comfortable in measurement.

When central limitation theorem holds, the random process $p(t,\vec{r})$ is supposed to be Gaussian, and, in doing so, to get a complete statistical description of scalar field $p(t,\vec{r})$ one needs just spatial correlation function available, see Eq. 6. In steady random field $p(t,\ \vec{r})$ the correlation function takes the form

$$R_{p^2}(t_1,t_2,\vec{r}_1,\vec{r}_2) = R_{p^2}(\tau,\vec{r}_1,\vec{r}_2), \tag{7}$$

where $\tau = t_1 - t_2$, and in the given case R_{p^2} is solely function of $t_1 - t_2$. At $\tau = 0$ and $\vec{r}_1 = \vec{r}_2$ $R_{p^2}(\tau = 0,\ \vec{r}_1)$ is named autocorrelation function, i.e. the potential energy density of random acoustic field at point $\vec{r}_1$. The function in Eq. 7 can be characterized by cross-spectral density, Ref. 7,

$$S_{p^2}(f;\vec{r}_1,\vec{r}_2) = \int R_{p^2}(\tau;\vec{r}_1,\vec{r}_2)e^{-j2\pi f\tau}d\tau, \tag{8}$$

that is the Fourier transform of R_{p^2} over τ.

In homogeneous field in which the correlation function only depends on $\vec{r}_1 - \vec{r}_2 = \vec{r}$, i.e. the difference between the measurement points cross-correlation spectral density has the form

$$S_{p^2}(f;\vec{r}_1,\vec{r}_2) = S_{p^2}(f;\vec{r}). \tag{9}$$

Isotropic field with spectral density written as Eq. 1.27 is an important particular case of a homogeneous field with spectral density

$$S_{p^2}(f;\vec{r}) = S_{p^2}(f,0)\,\frac{\sin(kr)}{kr}, \tag{10}$$

where $S_{p^2}(f,0)$ is the spectral density at $|\vec{r}|=0$, $k=2\pi f/c$ is the wave number, and c is the sound speed.

The basic property of isotropic field is the uniform angular distribution of the energy density. Isotropic field coherence function, Ref. 8,

$$\gamma_{12}^2(f) = \left(\frac{\sin kr}{kr}\right)^2. \tag{11}$$

For a given wave number $k_0 = 2\pi/\lambda_0$, the first zero in the isotropic noise coherence function curve has the ordinate r equal to a half wavelength of the sound $\lambda_0/2$. Thus, to suppress the isotropic noise at a given frequency f_0, the hydrophones are spaced by $\lambda_0/2$ in hydroacoustic detection systems.

The combined receiver measures four acoustic field characteristics, the acoustic pressure $p(t)$ along with three orthogonal components of the particle velocity vector

$\vec{V}(t)\{V_x(t), V_y(t), V_z(t)\}$ or the particle acceleration vector $\vec{a}(t)\{a_x(t), a_y(t), a_z(t)\}$. In each deployment the Cartesian axes assigned to the combined receiver employed were arranged the following way, z-axis was vertical downward from the surface to the bottom, and x- and y-axes laid in the horizontal plane. In drifting combined sensor systems x-axes used to be directed down the near-surface wind.

Let us suppose $p(t)$, $V_x(t)$, $V_y(t)$, $V_z(t)$, and $a_x(t)$, $a_y(t)$, $a_z(t)$ to be steady ergodic processes with zero means. The measurements are made at a given point in space, and the acoustic field components data are random functions of time.

Consider the following correlation functions in a given frequency band Δf,

1. Autocorrelation function of the acoustic pressure $p(t)$,

$$R_{p^2}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T p(t)p(t+\tau)dt\,; \tag{12}$$

2. Correlation function of the particle velocity components,

$$R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T V_i(t)V_j(t+\tau)dt\,,\ \text{where } i,j = x,y,z \tag{13}$$

3. Cross-correlation function,

$$R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T p(t)V_j(t+\tau)dt\,,\quad \text{where } i = p(t),\, j = x,y,z \tag{14}$$

For steady ergodic processes at $\tau = 0$, the statistical expressions in Eqs. 12-14 are physically,

1. Eq. 12 represents the potential energy of the field

$$R_{p^2}(\tau = 0) = E_p\,. \tag{15}$$

2. Eq. 13 repesents the kinetic energy components

$$R_{ij}(\tau = 0) = E_{ki} \text{ for } i = j = x = y = z, \tag{16}$$

$$R_{ij}(\tau = 0) = \Pi_{ij}, \quad \text{for } i \neq j, \text{ where } \Pi_{ij} \text{ is the impulse component.} \tag{17}$$

3. Eq. 14 presents the orthogonal components of the energy flux density vector

$$R_{ij}(\tau = 0) = \{I_x = pV_x, I_y = pV_y, I = pV_z\} \qquad \text{where } i = p,\, j = x,y,z. \tag{18}$$

Eqs. 12-14 can be generalized for complex random processes. Once $p(t)$, $V_x(t)$, $V_y(t)$, and $V_z(t)$ are supposed to be complex random centered processes, the following quantities can be written,

$$R_{p^2}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T p(t)p^*(t+\tau)dt\,,$$

$$R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T V_i(t)V_j^*(t+\tau)dt\,,\ \text{where } i,j = x,y,z, \tag{19}$$

$$R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T p(t)V_j^*(t+\tau)dt\,,$$

where $i = p(t)$, $j = x,y,z$, and $*$ represents the complex conjunction operation.

That is, the correlation functions from Eqs. 12-19 completely describe spatial properties of the acoustic field energetics at the measurement point.

2. SPECTRAL RELATIONSHIPS

Representation of spatial properties of the acoustic field energetics via its autospectra and cross-spectra provides great scope for complex acoustic fields examination, especially in detecting spectral components, tones as well as noise-like signals produced by various sources and present in underwater ambient noise.

While using FFT, no preliminary autospectra or cross-correlation functions are required to calculate spectra of interest, i.e. time series of a random process are to be directly transformed into frequency representation. Fourier components of random functions *p(t), $V_x(t)$, $V_y(t)$*, and *$V_z(t)$* are determined as

$$p_k(f,T)=\int_0^T p_k(t)e^{-i2\pi ft}dt\,,\qquad V_{k,j}(f,T)=\int_0^T V_{k,j}(t)e^{-i2\pi ft}dt\,, \tag{20}$$

where k is the number of transformations of time series T seconds long.

One-side cross-spectral and autospectral densities are determined as follows

$$\begin{aligned}
S_{pV_j}(f)&=\lim_{T\to\infty}\frac{2}{T}\left\langle p_k^*(f,T)V_{k,j}(f,T)\right\rangle,\\
S_{V_iV_j}(f)&=\lim_{T\to\infty}\frac{2}{T}\left\langle V_{k,i}^*(f,T)V_{k,j}(f,T)\right\rangle,\ (i\neq j),\\
S_{V_i^2}(f)&=\lim_{T\to\infty}\frac{2}{T}\left\langle \left|V_{k,i}(f,T)\right|^2\right\rangle,\\
S_{p^2}(f)&=\lim_{T\to\infty}\frac{2}{T}\left\langle \left|p_k(f,T)\right|^2\right\rangle,
\end{aligned}\qquad \text{where } i,j=x,y,z. \tag{21}$$

The $S_{pV_i}(f)$, $S_{V_iV_j}(f)$, $S_{V_i^2}(f)$, $S_{p^2}(f)$ spectra are identical to appropriate ones calculated from correlation functions of Eq. 12-14, Ref. 8.

Cross-spectral quantities $S_{pV_i}(f)$ and $S_{V_iV_j}(f)$ are complex quantities.

The cross-spectrum

$$S_{pV_i}(f)=C_{pV_i}(f)+iQ_{pV_i}(f)=\left\langle S_{V_i}(f)S_p^*(f)\right\rangle=\left\langle\left|S_p(f)\right|\left|S_{V_i}(f)\right|\right\rangle\cos<\varphi_{pV_i}>+$$

$$+i\left\langle\left|S_p(f)\right|\left|S_{V_i}(f)\right|\right\rangle\sin<\varphi_{pV_i}>,\qquad (i=x,y,z),$$

where $C_{pV_i}(f)=\left|S_{pV_i}(f)\right|\cos<\varphi_{pV_i}(f)>$ and $Q_{pV_i}(f)=\left|S_{pV_i}(f)\right|\sin<\varphi_{pV_i}(f)>$ are real functions.

The cross-spectrum magnitude

$$\left|S_{pV_i}(f)\right|=\left(C_{pV_i}^2(f)+Q_{pV_i}^2(f)\right)^{1/2}. \tag{23}$$

The phase spectrum

$$\varphi_{pV_i}(f)=\text{arc tn}\left[Q_{pV_i}(f)\big/C_{pV_i}(f)\right]=\text{arc t n}\left[\text{Im S}_{pV_i}(f)\big/\text{Re }S_{pV_i}(f)\right], \tag{24}$$

were Re and Im denote real and imaginary parts of the complex function $S_{pV_i}(f)$, *i = x,y,z.*

3. TYPICAL SINGLE-POINT COHERENCE FUNCTION

Regular coherence function, Ref. 8, is used to be employed in our studies. Since simultaneous measurements of random functions *p(t), $V_x(t)$, $V_y(t)$,* and *$V_z(t)$* are made simultaneously at a given point in space, we will use the term "regular single-point coherence function".

The following coherence functions will be employed,

$$\gamma^2_{pV_i}(f) = \frac{\left|S_{pV_i}(f)\right|^2}{S_{p^2}(f) S_{V_i^2}(f)}, \quad 0 \le \gamma^2_{pV_i} \le 1, \tag{25}$$

$$\gamma^2_{V_iV_j}(f) = \frac{\left|S_{V_iV_j}(f)\right|^2}{S_{V_i^2}(f) S_{V_j^2}(f)}, \quad 0 \le \gamma^2_{V_iV_j} \le 1, \tag{26}$$

The coherence function is similar to quadrature of the normalized correlation function at a given frequency. Physically the coherence function in Eq. 25 is the quadrature of normalized acoustic field intensity at a given frequency. It looks more convenient and carries more information than the correlation function does; this being so, the coherence function will be mostly employed while doing noise analysis in vector measurements, other than the coherent function.

As seen from Eqs. 22-26, in the case of determined traveling wave in x-axis direction, $\gamma^2_{pV_x}(f) = 1$, since $\varphi_{pV_x}(f) = 0°$. In doing so, the *p(t)* and $V_x(t)$ processes are supposed to be coherent. In an event of standing wave in x-direction, the *p(t)* and $V_x(t)$ processes are coherent as well and $\gamma^2_{pV_x}(f) = 1$, since $\varphi_{pV_x}(f) = 90°$.

The coherence function $\gamma_{pV_x}(f) = 0°$ when $< \cos\varphi_{pV_x}(f) > = < \sin\varphi_{pV_x}(f) > = 0$; this thing takes place for out-of-phase, i.e. incoherent *p(t)* and $V_x(t)$ processes only.

The coherence function is less than unit and greater than zero due to the following reasons,

inter-relationship between random processes *p(t)* and $V_x(t)$ are not proportional;

the data acquired are affected by external noise;

According to Virial Theorem, see Ref. 2, proportional relationships between *p(t)* and $V_x(t)$ are to be held in the mean in the acoustic field. In doing so, while having $0 \le \gamma^2_{pV_x}(f) \le 1$, one can conclude a coherent component contribution to the ambient noise acoustic field. Additional information on the nature of the coherent component can be inferred from the shape of the phase spectrum $\varphi_{pV_i}(f)$, Eq. 24.

Using the notion of coherent output power $S_{coh}(f)$ as a part of the acoustic field power related to linear proportion between *p(t)* and $V_x(t)$, one can write

$$S_{coh,i}(f) = \gamma^2_{pV_i}(f) S_{p^2}(f). \tag{27}$$

Thus, the remained spectrum related to incoherent diffusive component of the acoustic field will take a shape

$$S_{dif,i}(f) = \left[1 - \gamma^2_{pV_i}(f)\right] S_{p^2}(f), \tag{28}$$

for a given direction *i = x,y,z.*

4. SPECTRAL ANALYSIS IN VECTOR ACOUSTICS

A single four-component combined receiver provides simultaneous time series of the acoustic pressure *p(t)* and three orthogonal components of the particle velocity vector $\vec{V}(t)\{V_x(t), V_y(t), V_z(t)\}$ at a given point in space. In the deployment time series *p(t),* $V_x(t)$,

$V_y(t)$, and $V_z(t)$ chosen to be processed satisfied the requirements of homogeneity, stability, ergodicity and were centered random processes. The combined receiver axes were used to be oriented in physically important directions within the ocean waveguide, i.e. x- and y-axes lay in the horizontal plane with the x-axis directed down the average wind speed vector, and z-axis directed vertically down from the surface to the bottom. The specific arrangement of Cartesian axes assigned to the combined receiver enables to estimate properties of the ambient noise field anisotropy using spectral characteristics available.

Statistical analysis here is based on the following spectra,

1. $S_{P^2}(f)$, the acoustic pressure autospectrum from the output of a single unidirectional hydrophone incorporated in the combined receiver;
2. $S_{V_x^2}(f)$, $S_{V_y^2}(f)$, $S_{V_z^2}(f)$, autospectra of orthogonal particle velocity components from the vector sensor output;
3. $|S_{PV_x}(f)|$, $|S_{PV_y}(f)|$, $|S_{PV_z}(f)|$, magnitudes of the cross spectra along with corresponding phase spectra $\Delta\varphi_x(f)$, $\Delta\varphi_y(f)$, $\Delta\varphi_z(f)$, of random processes $p(t)$, $V_x(t)$, $V_y(t)$, and $V_z(t)$;
4. $\gamma^2_{PV_x}(f)$, $\gamma^2_{PVy}(f)$, $\gamma^2_{PV_z}(f)$, the coherence functions related to orthogonal x-, y-, and z-axes;
5. $|S_{V_xV_y}(f)|$, $|S_{V_xV_z}(f)|$, $|S_{V_yV_z}(f)|$, cross-spectral magnitudes with related phase spectra $\Delta\varphi_{xy}(f)$, $\Delta\varphi_{xz}(f)$, $\Delta\varphi_{yz}(f)$, and the coherence functions $\gamma^2_{V_xV_y}(f)$, $\gamma^2_{V_xV_z}(f)$, $\gamma^2_{V_yV_z}(f)$ of random processes $V_x(t)$, $V_y(t)$, $V_z(t)$;

In fact, the spectra grouped together in 2-4 represent the angular spectra in orthogonal directions *x, y,* and *z*. Combining the spectra from groups 2-4, one can calculate 3-component angular spectrum in the "intensity-frequency-angle" coordinate system, or the angular phase spectrum in the "phase difference-frequency-angle" coordinate system. Such angular spectra provide the complete representation of anisotropic and diffusive properties of the ambient noise as well as characteristics of both directivity and energetics of the signals to be found against the background noise.

Since the ambient noise diffusive component energy flux density must be equal to nil, one can separate the net noise field energy density into the energy density involved in the energy transport within the ocean waveguide and the energy density "frozen" in the waveguide. As the experiment evidences, over the entire 6 to 1000 Hz frequency band under investigation, a large share of the dynamic ambient noise energy density contributes to the diffusive field, whereas it is not necessarily so for distant shipping noise. The challenge of extreme importance now is to eliminate the diffusive contribution from the net energy density while doing spectral estimation of the coherent component of the ambient noise or a signal mixed with additive noise. An attempt to develop such technique for spectral estimation using the acoustic pressure measurements has been made by Pisarenko, see Ref. 9, using harmonic expansion technique.

Pisarenko technique like any other innovative method of spectral estimation, Ref. 9, makes an attempt to better spectral resolution and signal detection comparatively to Fast Fourier Transform (FFT) technique. In fact, this is an attempt to estimate a share of additive uncorrelated noise using autocorrelation function and subsequent subtraction of the resultant share from the net energy density in the process calculated from the acoustic pressure data. However, the Pisarenko technique lacks rigorous criterion to determine the share of interest that can result in excessive estimation of the noise compensation. Nevertheless, such technique appears to be prospective for it permits to find out the spectral structure of the modified correlation function supposed to be a sum of tones. Fig.1 shows normalized spectra of two 3- and 4-Hz tones against the background of additive white noise with dispersion of 10% of the signal power. Spectral estimation was made by using Blackman-Tiuckey technique (Fig.1,a), by autoregressive technique (Fig.1,b), and by Pisarenko technique (Fig.1,c). As seen from Fig.1, the Pisarenko technique has the best resolution providing two tones as delta functions against the white noise background, and yet the requirements imposed upon calculations related to most innovative techniques employed in spectral estimations are more complicated than that in FFT-based data processing offering a little promise for employment in real time analysis systems, though may be prospective for possible future research, see Refs. 7 and 9.

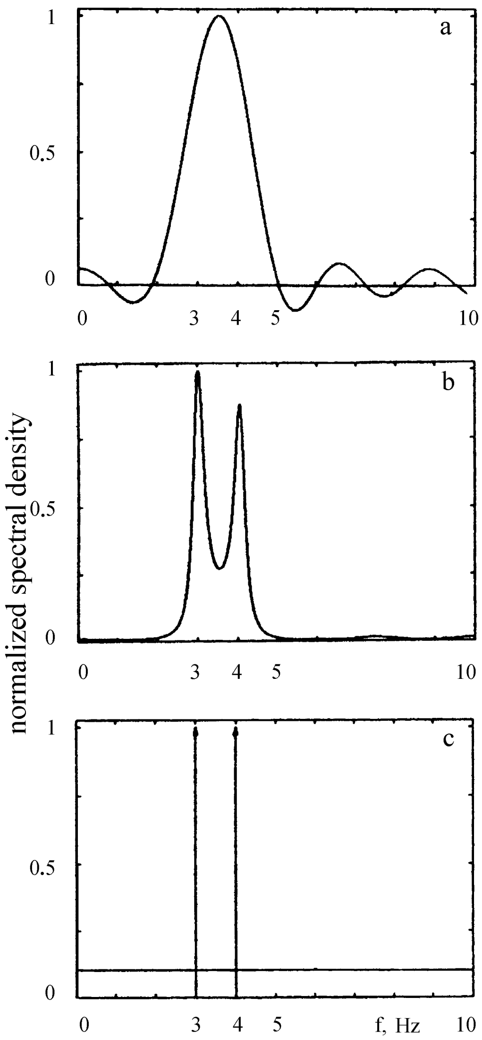

Fig.1 Three spectral estimates made for superposition of 3- and 4-Hz tones, and additive white noise, the noise dispersion is 10% of the signal power, a) Blackman-Tiuckey technique application, b) autoregressive estimate, and c) estimate made by harmonic expansion offered by Pisarenko. From Ref. 9

While doing FFT-based processing of combined receiver data to obtain spectral estimates of the ambient noise energy flux density, they never have the diffusive ambient noise contribution as directly follows from the definition of the energy flux density vector as a physical quantity. That essentially betters signal detection against the ambient noise background while imposing no any additional requirement on FFT-based data procession.

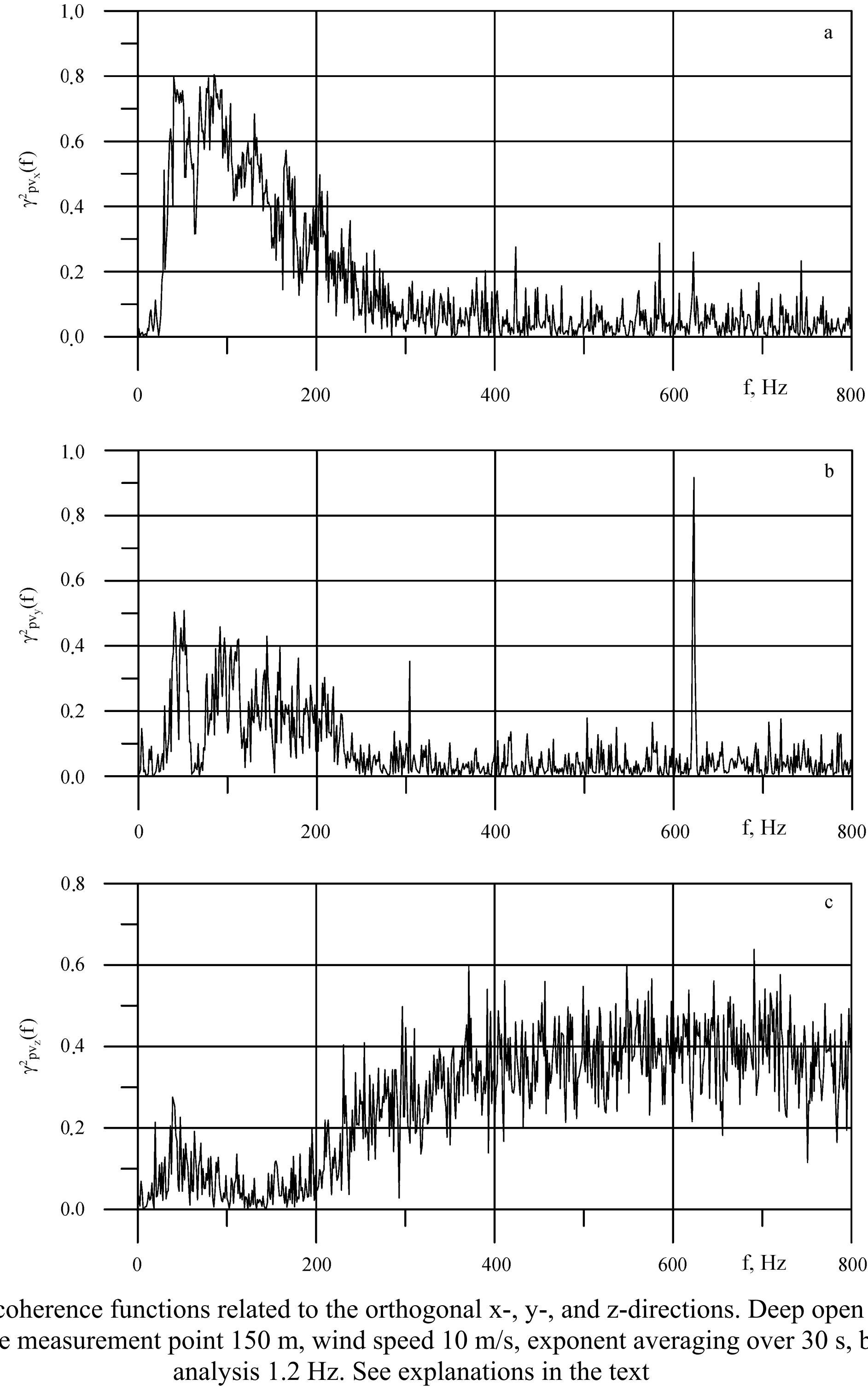

Fig.2 The coherence functions related to the orthogonal x-, y-, and z-directions. Deep open ocean, depth of the measurement point 150 m, wind speed 10 m/s, exponent averaging over 30 s, band of analysis 1.2 Hz. See explanations in the text

By way of illustration, Figs.2 and 3 show the coherence function and phase spectra of underwater ambient noise in the deep ocean. As follows from these plots, the underwater ambient noise field in the deep ocean is anisotropic in both horizontal and vertical planes. The extent of anisotropy is a function of frequency. The relationship between the coherent and diffusive components of the total field is also frequency-dependent function. The 622-Hz tone is clearly visible against the background of virtually completely diffusive field of the dynamic noise, Fig.2,b. In the horizontal plane anisotropic noise produced by distant shipping is clearly visible up to 300 Hz, Fig.2,a,b, whereas it is not found in the vertical spectrum $\gamma^2_{PV_z}(f)$, Fig.2,c. Directivity of coherent fluxes is clearly seen from the phase spectra $\Delta\varphi_{xy}(f)$, $\Delta\varphi_{xz}(f)$, and $\Delta\varphi_{yz}(f)$, Fig.3.

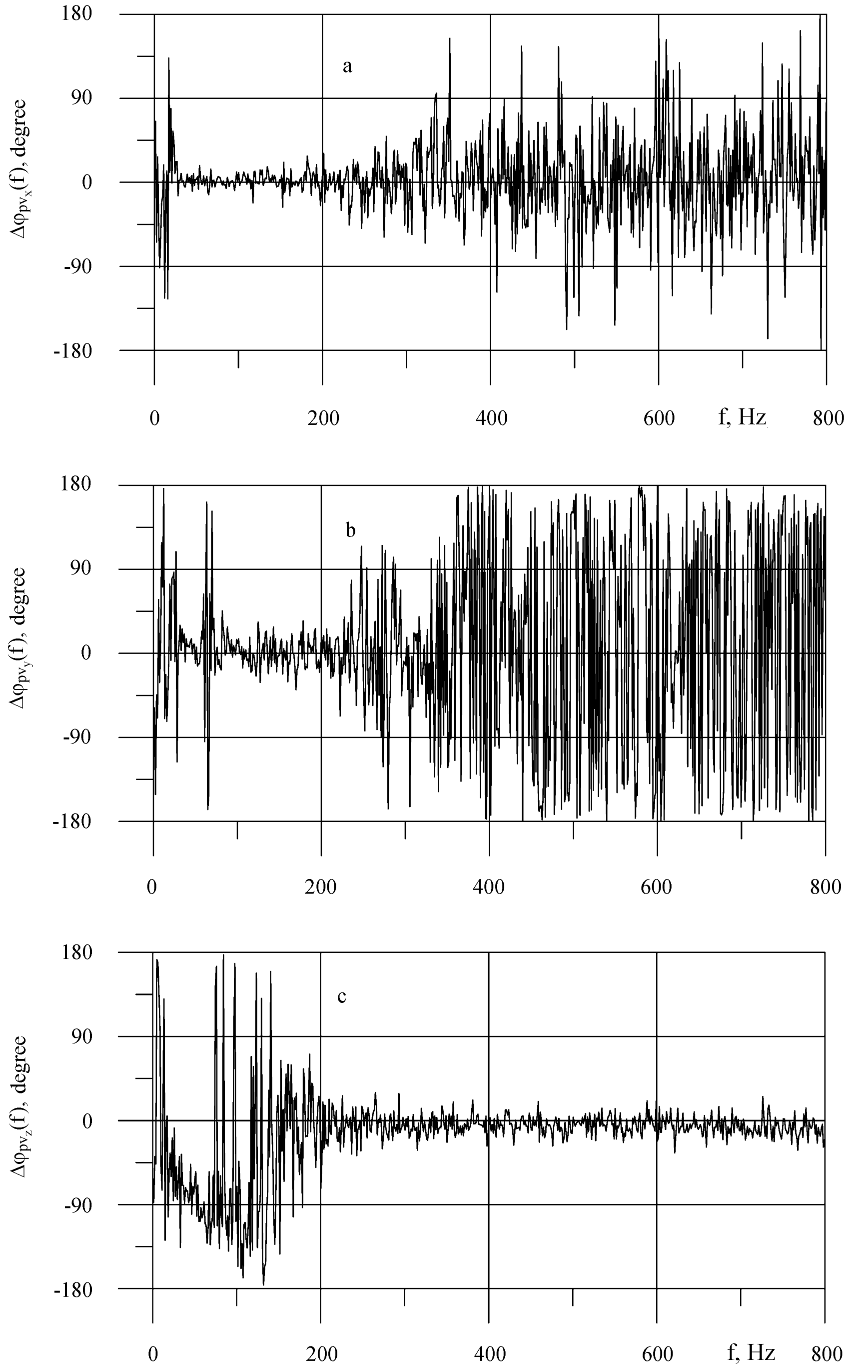

Fig.3 Phase spectra related to Fig.2. The conditions are the same as in Fig.2

As seen from Eqs.22-25, 28 and Figs.2 and 3, the magnitude of the diffusive field energy density dictates the shape of the coherence function, whereas never affects the phase spectra.

At the frequencies up to 300 Hz the distant shipping intensity, see Figs.2 and 3, varies over the deployment region, whereas in the 300 to 800 Hz frequency band of the dynamic noise the properties of the cross spectra, the coherence functions, and the phase spectra remain the same mostly depending on meteorological conditions and the depth of the measurement point. In the vertical plane between 300 and 800 Hz Figs.2 and 3 evidence an existence of the noise energy flux with nonzero coherence function $\gamma^2_{PV_z}(f) \approx 0.3\text{-}0.4$; whereas in the horizontal plane in x-direction there is another dynamic noise energy flux and in y-direction the diffusive noise field is observed. Using general properties of underwater ambient noise, one can perfect mathematical and computational aspects of processing data collected by passive observation means.

5. CONCLUSIONS

On the base of properties of four random functions being the four acoustic field components, the acoustic pressure *p(t)* and three orthogonal components of the particle velocity vector $\vec{V}(t)$ $\{V_x(t), V_y(t), V_z(t)\}$ each supposed to be a homogeneous, stable, ergodic and centered time-function, the author have employed the FFT technique to estimate spectral and cross-spectral characteristics of the four components of the acoustic field.

The cross-spectral estimates are, in fact, directional characteristics of the acoustic field energetics. Having in hand directional spectral characteristics of the energetics related to orthogonal *x-, y-,* and *z*-directions, one can calculate the frequency-angular energy spectrum in each given direction that provides the complete spatial description of the ambient noise-and-signal acoustic field energetics.

The following vector properties of the ambient noise are of great importance,

-once the time shift is zero, the acoustic field energy flux density is equal to the cross-correlation function of the acoustic pressure and the particle velocity vector;

-the diffusive field energy flux density is universal nil;

These vector properties of the acoustic field can be successfully used in the development of innovative progressive techniques capable of estimating spectral characteristics of the ambient noise energetics, and most effective methods for signal detection against the noise background.

REFERENCES

[1] M. Isakovich, General acoustics, Nauka, 495, Moscow 1973.

[2] L. Landau, E. Lifshits, Hydrodynamics, Nauka, 736, Moscow 1986.

[3] N. Umov, On the energy motion in bodies, Dis, Odessa 1873.

[4] M. Born, E. Wolf, Fundamentals of optics, Nauka, 855, Moscow 1970.

[5] N. Maltzev, Acoustics of the ocean medium, 4-10, Nauka, Moscow 1989.

[6] G. Birkhoff, Computational ocean acoustics, The invited lectures from workshop held at Yale University, Aug. 1-3, 1984, Comput. and Math. Appl., Vol. 7/8, 891, 1985.

[7] S. Cay, S. Marple, Modern methods in spectral analysis, TIIER, Vol. 69, №11, 5-51, 1981.

[8] G. Bendat, A. Pirsol, Engineering applications of correlation and spectral analysis, Mir, 321, Moscow 1983.

[9] V. Pisarenko, On the estimation of spectra by means of nonlinear functions on the covariance matrix, Geophys. J. Roy. Astron. Soc., Vol. 28, 511-531, 1972.
[10] G. Mc.Lellan, Polidimensional spectral analysis, TIIER, Vol. 70, №9 139-152, 1982.
[11] V. Shchurov, Vector acoustics of the ocean, Dalnauka, 307, Vladivostok 2003.