A HYBRID OPTIMIZATION METHOD FOR DESIGNING POLYPHASE SEQUENCES

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Polyphase sequences which have a low autocorrelation ensure an easily detectable peak in the output of a matched filter of a radar receiver. The low autocorrelation for polyphase codes is usually described in terms of the maximum magnitude of its sidelobes level. In this paper, an evolutionary algorithm combined with a local optimizer is used to search for polyphase codes with a small sidelobe level of an aperiodic autocorrelation function. The evolutionary algorithm is based on a floating-point representation and the Gaussian mutation is used to produce offspring for the next generation. The local optimizer is applied to find polyphase sequences which are good starting points for the evolutionary algorithm. This research demonstrates that optimization methods can effectively find polyphase codes with the low autocorrelation and seems to be very promising for future research in area of computer optimization for radar polyphase codes synthesis.

INTRODUCTION

Phase coding is one of the early techniques for pulse compression of radar signals. The advantage of the pulse compression method is the increase of the average transmission power while retaining the range resolution corresponding to a short pulse. Pulse compression may be performed by means of matched filter, in other words, by correlating the received signal with a stored replica of the transmitted signal.

The aim of this research was to develop an evolutionary algorithm combined with a local optimizer and apply this hybrid technique to solve a very difficult real-world problem of search for polyphase sequences, which have the most desired properties for radar applications.

1. PROBLEM DEFINITION

The phase-coded pulse, shown in Fig. 1, is divided into M bits of identical duration t_b=T/M, and each bit is assigned with a different phase value φ_m. Such sequence of phase values is an example of a potential solution for the optimization problem of designing polyphase code.

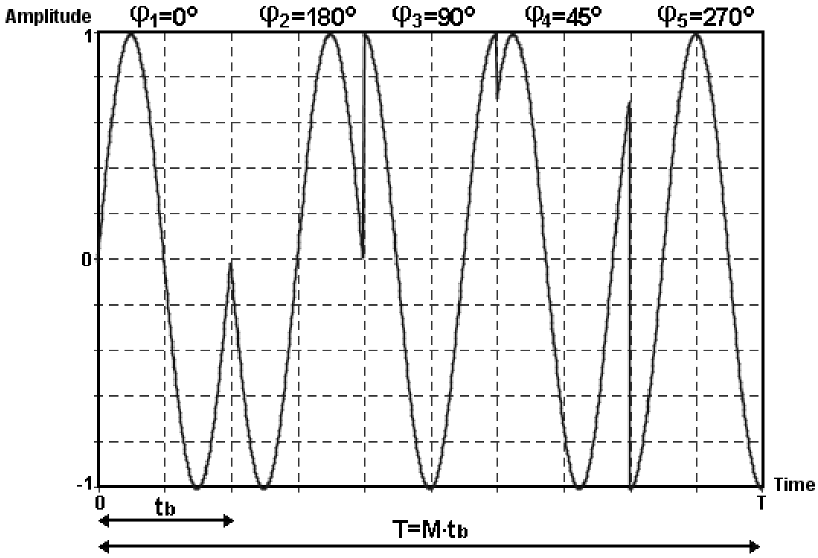

Fig.1 An example of the phase-coded pulse

The complex envelope of such phase-coded pulse is given by

$$u(t) = \frac{1}{\sqrt{T}} \sum_{m=1}^{M} e^{j\varphi_m} rect\left[\frac{t-(m-1)\cdot t_b}{t_b}\right]. \tag{1}$$

The pulse compression goodness of a polyphase code is based on its autocorrelation function. The Autocorrelation function of phase-coded pulse is a continuous function of the delay τ and is defined by

$$R(\tau) = \int_{-\infty}^{\infty} u(t)\cdot u^*(t-\tau)dt\,, \tag{2}$$

where the asterisk denotes the complex conjugate. This function may be expressed as the discrete aperiodic autocorrelation function according to the interpolation done in the complex plane and given by equation

$$R(\tau) = R(k\cdot t_b + \eta) = \frac{1}{t_b\cdot M}\left[(t_b-\eta)\cdot R[k] + \eta\cdot R[k+1]\right], \tag{3}$$

where R[k] is the discrete aperiodic autocorrelation function evaluated at $\tau = k$ and $0 \le \eta < t_b$ [2]. An example of the autocorrelation function is shown in Fig. 2.

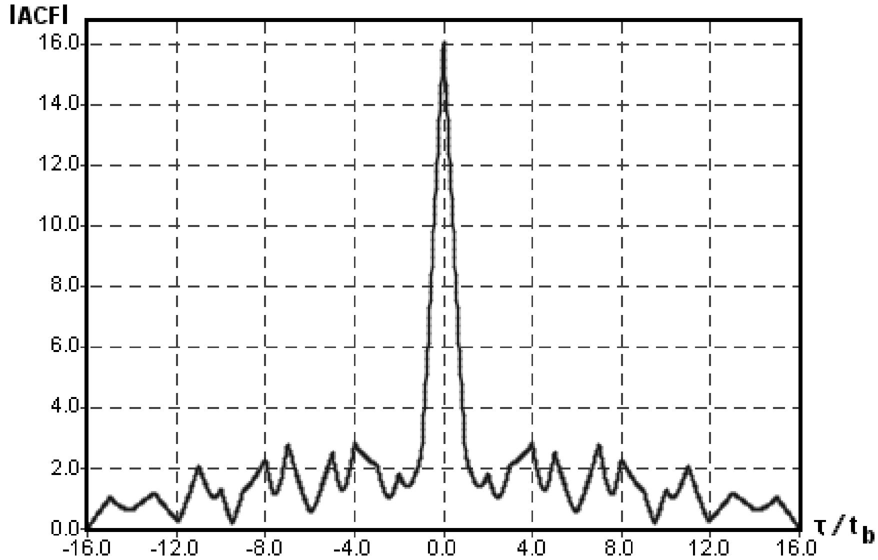

Fig.2 An example of the autocorrelation function

For examining the properties of polyphase sequences, it is sufficient to calculate the autocorrelation function only at integer multiples of the bit duration. The aperiodic autocorrelation coefficient c_k may then be written as

$$c_k = \sum_{m=1}^{M-k} a_m \cdot a_{m+k}^{*}, \;\; k = -(M-1),....,(M-1), \tag{4}$$

where the asterisk denotes the complex conjugate and $a_m = e^{j\varphi_m}$, for $1 \le m \le M$, $0 \le \varphi_m < 2\pi$. Because the autocorrelation function is symmetrical with respect to its mainlobe $|c_k|=|-c_k|$, it may be rewritten with positive index k

$$c_k = \sum_{m=1}^{M-k} e^{j(\varphi_m - \varphi_{m+k})}, k = 0,....,(M-1)\,. \tag{5}$$

A low autocorrelation for codes is usually described in terms of the maximum magnitude of its sidelobes level. Polyphase sequences, which have low sidelobe levels, ensure an easily detectable peak in the output of a matched filter of a radar receiver.

Because first coefficient $|c_0|$ is the mainlobe and the last sidelobe equals one in any case $|c_{M-1}|=1$, the objective function of the optimization problem can be eventually expressed as follows

$$l_\infty(C) = \max\{|c_k| : 1 \le k \le M-2\}\,. \tag{6}$$

Summing up, the optimization goal is to find a polyphase sequence which has a maximum sidelobe level as low as it possibly can. The Problem under consideration is modeled as a nonlinear, NP-hard optimization problem in continuous variables and with numerous local optima [3].

2. OPTIMIZATION ALGORITHM

In this research, an evolutionary algorithm combined with a local optimizer was used to search for polyphase codes with a small sidelobe level of an aperiodic autocorrelation function.

The optimization algorithm is shown in flowchart form in Fig. 3. The evolutionary algorithm begins by initialising a population of potential solutions for the objective function. Next the local optimizer is applied to improve starting points. New solutions are then created by mutating those of the initial population. All solutions then have their “fitness” evaluated and a selection criterion is applied to remove worse solutions. This process is iterated using the selected solutions until the stopping criterion is met.

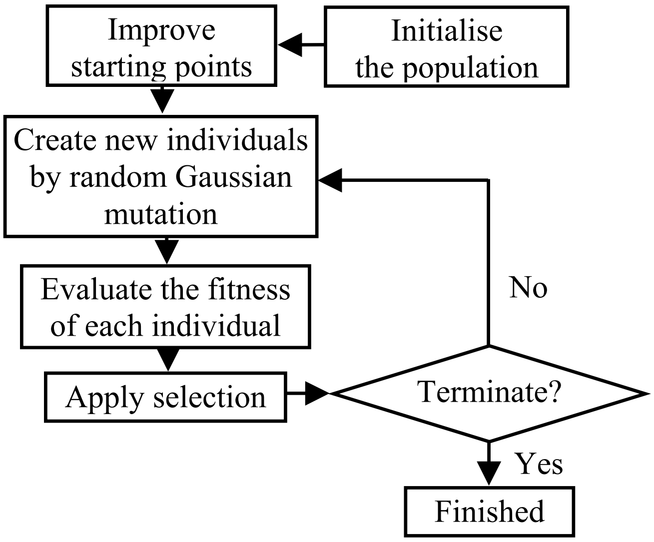

Fig.3 The evolutionary algorithm in flowchart form

For such nonlinear numerical optimization problem, the evolutionary algorithm was based on a floating-point representation. Each individual x in the population was represented as a vector of floating-point numbers $\mathbf{x}=(x_1, x_2, ..., x_n)$.

In order to produce offspring for the next generation, the Gaussian mutation was used. The crossover operator was rejected because of its disruptive influence on the convergence of the algorithm. Mutations were then realized by adding to each component of the vector a random Gaussian number with mean zero and standard deviation σ, decreasing during the evolutionary process and depending on the number of generations.

$$x_i^{t+1} = x_i^t + N(0, \sigma(t)), \quad for \quad \sigma(t) = 1 - 0.9 \cdot \sigma_{initial} \cdot \frac{t}{T}, \tag{7}$$

where t - current generation number, T – maximum generation number [6].

For the selection of individuals for the next generation, tournament selection was applied. In this approach, the individuals in the population are randomly grouped in pairs, the fitness levels of two individuals are then compared to each other. The individual with the better fitness survives to the next iteration while the other is terminated.

The local optimizer was applied to find polyphase sequences which were good starting points for the evolutionary algorithm. Such approach significantly improved the performance of the evolutionary algorithm. The local method utilized another evaluation function than the evolutionary algorithm, which was the sum of the sidelobe energies

$$l_2(C) = \sqrt{\sum_{k=1}^{n-1} |c_k|^2}, \tag{8}$$

but minima of both functions should lie close together.

Two dimensional examples of both functions utilized in the optimization process are shown in Fig. 4 and Fig. 5.

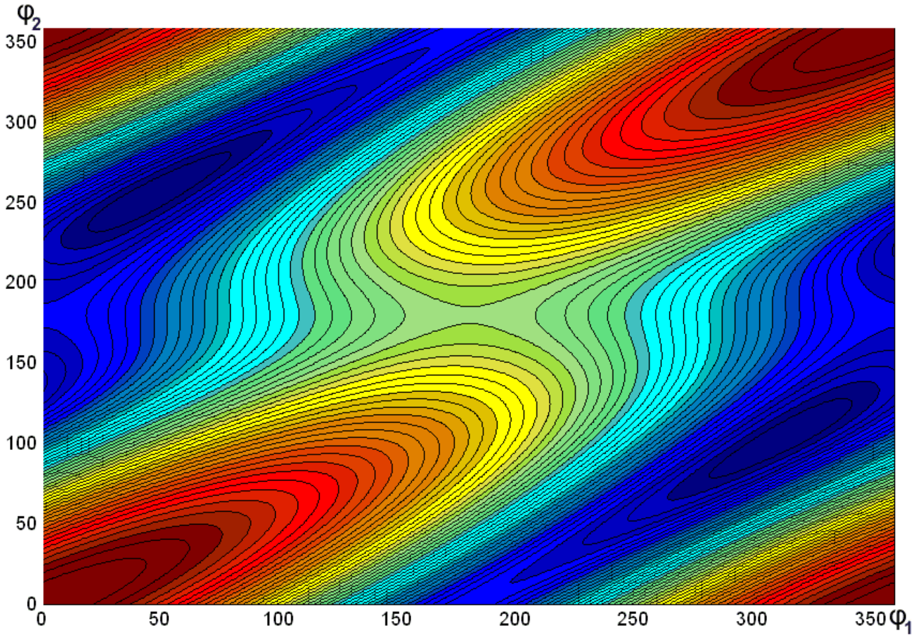

Fig.4 A two dimensional example of the objective function for the local method

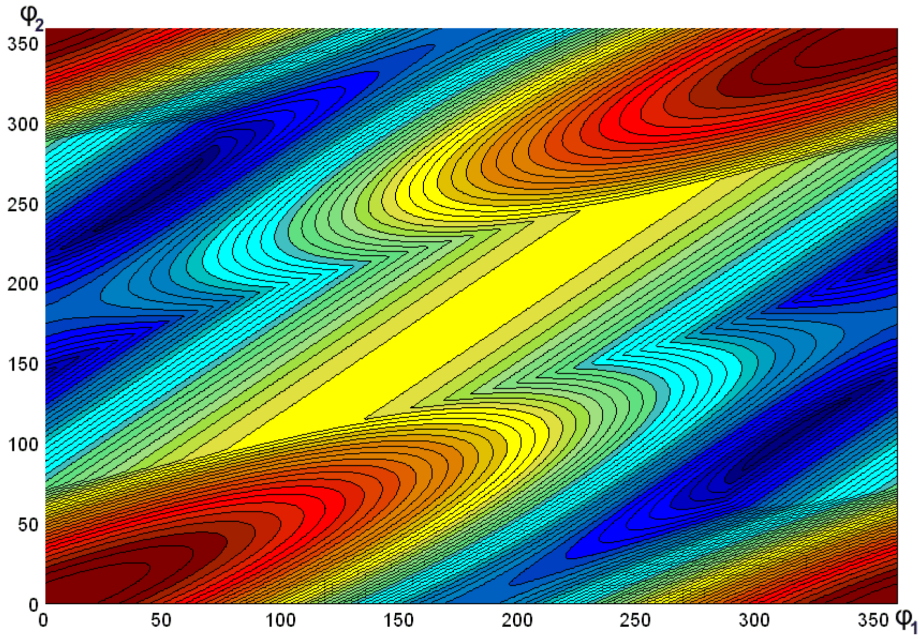

Fig.5 A two dimensional example of the objective function for the evolutionary algorithm

It is worth noticing that the base energy function (Fig. 4) has more regular surface and is easier to optimize. Briefly speaking, the aim of the local optimizer, which utilized equation (8), was to move some initial points towards regions suspected of containing outstanding solutions for the latter function (defined by equation (6)).

The local optimizer was based on the Hooke-Jeeves direct search algorithm. This algorithm consists of two steps. First exploratory moves are made about a base point solution to determine an appropriate direction of search. Then, in second step – pattern search, the base point solution is moved, according to the previously determined direction, to a new location. If in exploratory search, all trials are not found the better value of the function, the algorithm goes back to the best recent base point and then step size is reduced and exploratory moves are made again. These stages are repeated until a step size becomes less than a pre-set value [5].

3. RESULTS

This research demonstrates that evolutionary strategies can effectively find polyphase sequences with the low autocorrelation and seems to be very promising for the future research in area of computer optimization for radar polyphase codes design.

The evolutionary algorithm was tested for some arbitrary selected lengths from the range of 13 to 128. In some cases, especially for shorter sequences, the algorithm found a polyphase Barker sequence occurring when a value of maximum sidelobe level is less or equal to one [1,4]. In general, such sequence might be regarded as a perfect solution. In the remaining cases, the results were also satisfactory.

It's worth mentioning that the number of individuals in the population was ranging from one to three thousands and most of the results were obtained by only two or three runs of the algorithm therefore an influence of initial phase configurations on optimization results has not been studied yet.

The results are presented as values of Peak-to-Sidelobe Level ratio, which is often used to quantify the performance of sidelobe suppression. PSL is simply the ratio between the highest sidelobe and the mainlobe expressed in decibels.

The obtained results, illustrated in Fig. 6, range from 25.13 dB for the code length of 13 up to 34.47 dB for the code length of 128.

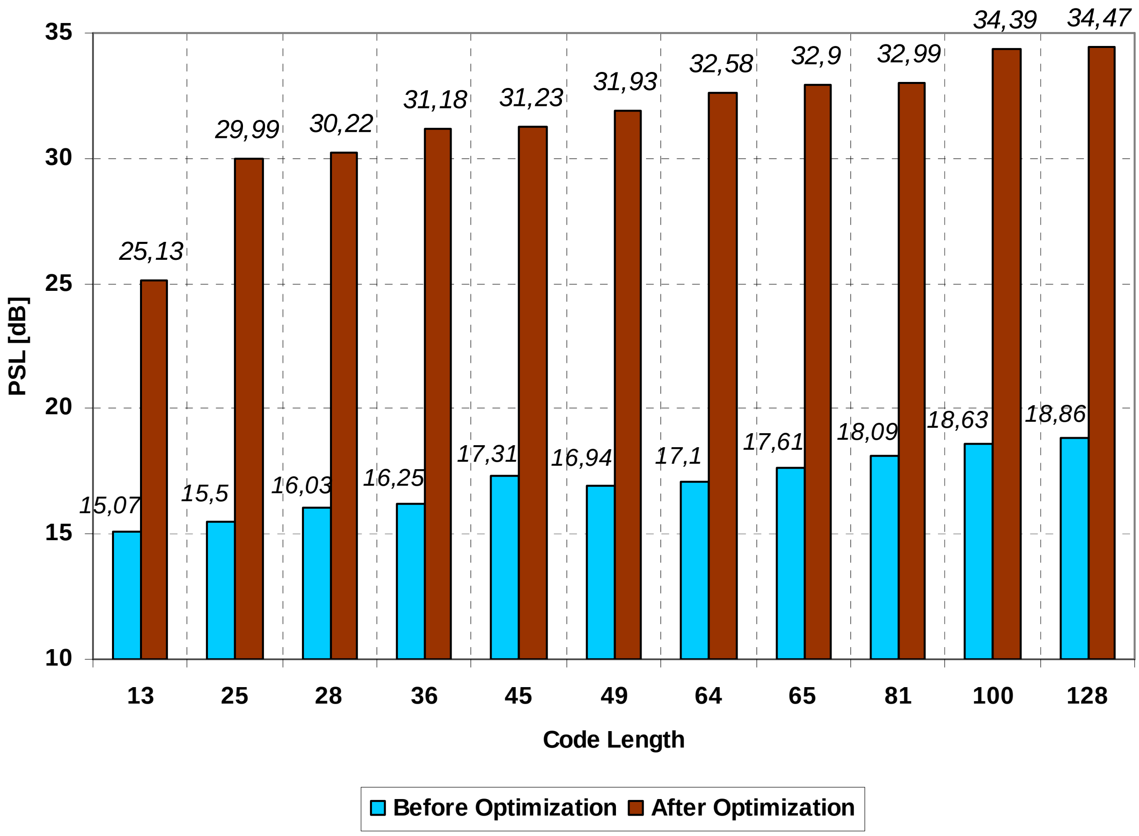

Fig.6 The results of the optimization process

The comparison of obtained codes with Frank and P4 sequences is shown in Fig.7. Comparing achieved results to the ones obtained by examining characteristics of well known Frank codes or P4 codes, we can easily notice that in every case for the same length the obtained codes has outperformed Frank and P4 codes.

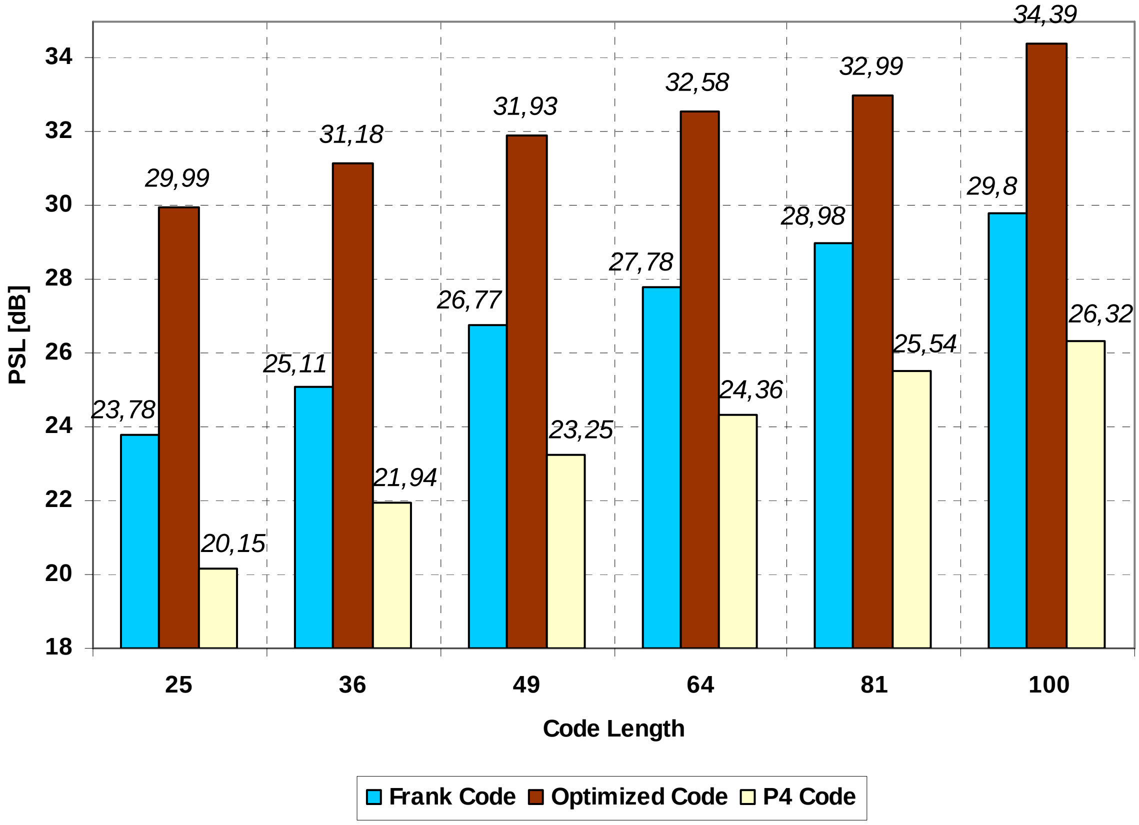

Fig.7 The comparison of Frank and P4 sequences with codes obtained in the optimization process

4. CONCLUSION

The results seem to be very promising for the future research in area of computer optimization for radar polyphase codes synthesis, especially considering the fact that all calculations were performed on a single standard computer.

The growth of the computing power should bring much better results, especially for longer sequences, and allow an optimization of very long codes. Therefore, future research will be oriented on implementation of parallel evolutionary algorithms using distributed programming. The improvement of the evolutionary algorithm and the local optimizer will also be considered.

It should be mention that Frank and P4 codes have higher Doppler tolerance than obtained sequences in the optimization process. Doppler resilience is particularly essential in cases where long pulses are transmitted against high velocity targets. Therefore, in future research, Doppler shift will be taken into consideration in the optimization process of polyphase signals. The problem of finding codes resistant to Doppler shift is very complicated and is based on examination of sidelobes in the ambiguity function of the polyphase signal. For example, the ambiguity function of a code obtained in this research with length 25 is shown in Fig. 8.

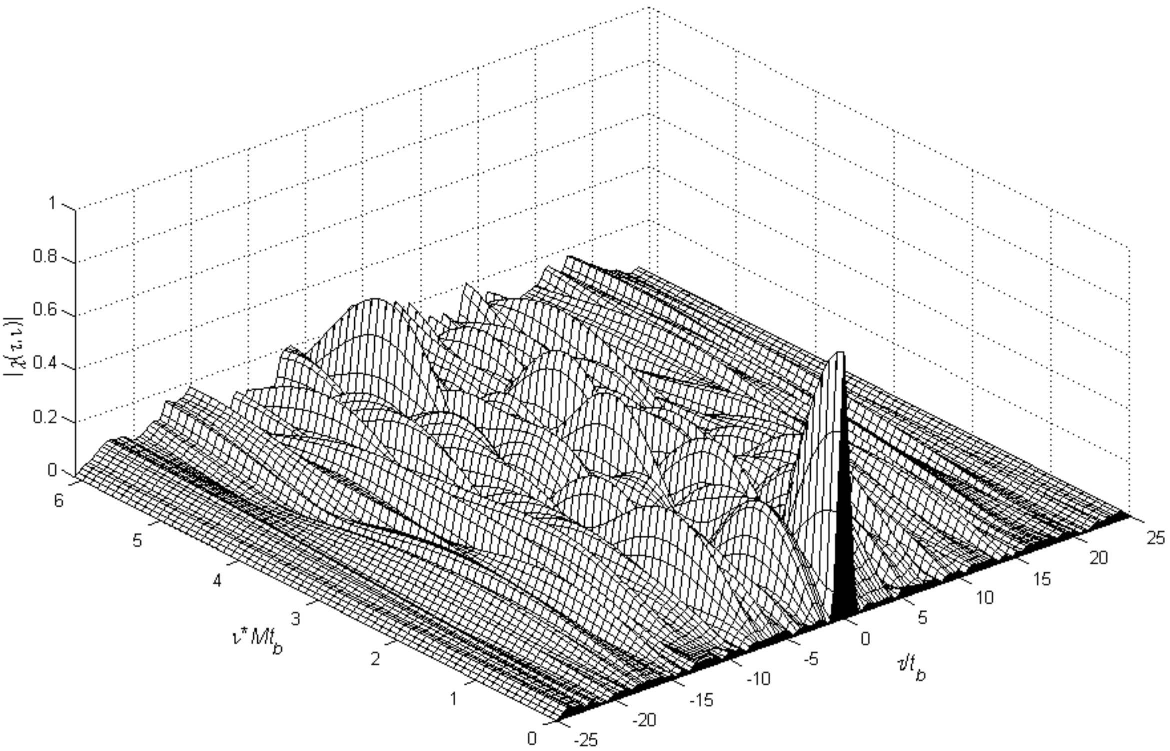

Fig.8 Partial ambiguity function of the 25-element code obtained in the optimization process

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