APPLICATION OF THE PERFECT COMBINATORIAL CONFIGURATIONS FOR CONFIGURE OF HIGH PERFORMANCE SONAR SYSTEMS

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Perfect combinatorial configurations can be used for finding optimal placement of structural elements in spatially distributed sonar or acoustic systems for configure high performance systems with non-uniform structure (e.g. arays of sonar antennas) with respect to positioning precision, and resolving ability, using novel design based on combinatorial configurations such as "Golomb rulers" and difference sets. These design techniques make it possible to configure sonar systems with fewer elements than at present, while maintaining or improving on resolving ability and the other significant operating characteristics of the system.

INTRODUCTION

Problem of structural optimization of radar or sonar systems relates to finding the best placement of its structural elements in spatially distributed systems as well as a better understanding of the role of geometric structure in the behavior of the systems. Research into underlying mathematical area involves the appropriate algebraic constructions based on finite groups in extensions of Galois fields [1].

Application of the algebraic constructions and modern combinatorial analysis provides optimal solution a lot of problems of high-resolution interferometry for radar, data communications, and signal design [2]. Some regular methods for constructing non-redundant two-dimensional n-element masks over $n \times n$ grids, based on algebraic combinatorial configurations such as difference sets are suggested in the publication [3]. However, the classical theory of combinatorial configurations can hardly be expected effective for constructing high-resolution 2-D or 3-D sonar systems, and finding optimal solutions for other problems in constructing radar and sonar systems including low side lobe sonar design and vector data coding. Hence, both an advanced theory and regular method for optimal solution the problems are needed.

1. PERFECT ORDERED CHAIN COMBINATORIAL CONSTRUCTIONS

The “ordered chain” approach to the study of elements and events is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a distributed technological system. It is the method of optimal structural proportions based on idea of “Perfect Numerical Bundle”. The simplest Perfect Numerical Bundles are "Ideal Golomb rulers" [2].

Here is an example of an Ideal Golomb ruler with n=3, namely 3-stage chain sequence {1,3,2} as being the Perfect Numerical Bundle (Fig.1).

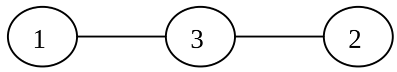

Fig.1 Perfect 3-stage ordered chain sequence

To see this, we observe, the all terms in each sum to be consecutive elements of the sequence allows an enumeration of all numbers from 1 to S_n = 6 exactly once:

1=1, 2=2, 3=3, 4=1+3, 5=3+2, 6=1+3+2.

Let us calculate all sums of the terms in the numerical n-stage ($n \geq 4$) chain sequence of distinct positive integers $C_n = k_1, k_2, \ldots, k_n$, where we require all terms in each sum to be consecutive elements of the sequence.

Clearly the maximum number of distinct sums is

$$S_n = 1+2+\ldots+n = n(n+1)/2 \quad (1)$$

For example, the maximum number of distinct sums of the 5-stage (n=5) chain sequence $\{k_1, k_2, \ldots, k_5\}$ is $S_{n=5} = 15$:

k_1	k_2	k_3	k_4	k_5
$k_1 + k_2$	$k_2 + k_3$	$k_3 + k_4$	$k_4 + k_5$	
$k_1 + k_2 + k_3$	$k_2 + k_3 + k_4$	$k_3 + k_4 + k_5$		
$k_1 + k_2 + k_3 + k_4$	$k_2 + k_3 + k_4 + k_5$			
$k_1 + k_2 + k_3 + k_4 + k_5$				

It is very hard problem to search distinct positive integers of the chain sequence $\{k_1, k_2, \ldots, k_5\}$, where we require all the all terms in each sum to be consecutive elements allows an enumeration numbers from 1 to S_n = 15 exactly once. Strictly speaking Ideal Golomb rulers of $n \geq 4$ not exist. So, it is possible to go on finding non-ideal Golomb rulers with $n \geq 4$ because well useful in applications to problems of signal design for radar, sonar, and data communications [2].

2. 1-D PERFECT ORDERED RING COMBINATORIAL CONSTRUCTIONS

If we regard the chain sequence C_n as being cyclic, so that k_n is followed by k_1, we call this a ring sequence. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length (number of terms) from 1 to n-1. In addition, there is the sum T of all n terms, which is the same independent of the starting point. Hence the maximum number of distinct sums S of consecutive terms of the ring sequence is given by

$$S = n(n-1)+1. \qquad (2)$$

Comparing the equations (1) and (2), we see that the number of sums S for consecutive terms in the ring topology is nearly double the number of sums S in the daisy-chain topology, for the same sequence C_n of n terms.

An n-stage sequence $C_n=\{k_1,k_2,...,k_n\}$ of natural numbers for which the set of all S circular sums consists of the numbers from 1 to $S = n(n-1)+1= n^2-n+1$ (each number occurs exactly once) is called an 1-D "Gold Numerical Ring" (GNR). Here is an example of an GNR with $n = 4$ and $S = 4^2-4+1=21$, namely {1,4,6,2} (Fig.2).

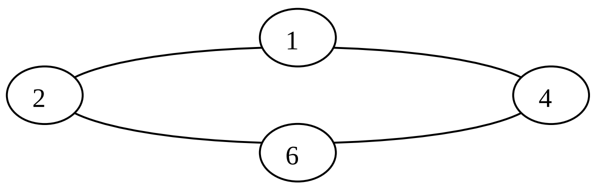

Fig.2 1-D GNR with parameters n=4 and R=1

To see this, we observe the table 1.

Tab.1 Sums of consecutive terms of the GNR with parameters n=4 and R=1

1=1	2=2	3=2+1	4
5=4+1	6=6	7=2+1+4	8=6+2
9=6+2+1	10=4+6	11=1+4+6	12=4+6+2

Note that if we allow summing over more than one complete revolution around the ring, we can obtain all positive integers as such sums. Thus:

13=1 + 4 + 6 + 2, 14=1 + 4 + 6 + 2 + 1, 15=2 + 1 + 4 + 6 + 2, 16= 2 + 1 + 4 + 6 + 2 + 1, etc.

Next, we consider a more general type of GNR, where the S ring-sums of consecutive terms give us each integer value from 1 to M, for some integer M, exactly R times, as well as the value M+1 (the sum of all n terms) exactly once. Here we see that:

$$M = n(n\text{-}1)/R \qquad (3)$$

An example of an GNR with *n*=4 and *R*=2 is the ring sequence {1,1,3,2}, for which the sums of consecutive terms are as follows (Fig.3).

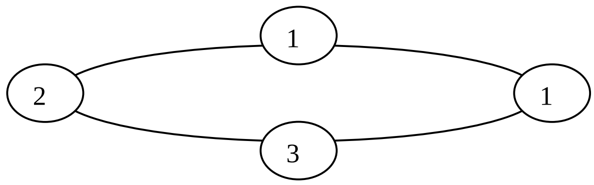

Fig.3 1-D GNR with parameters *n*=4 and *R*=2

There are calculated sums of consecutive terms (table 2).

Tab.2 Sums of consecutive terms of the GNR with parameters *n*=4 and *R*=2

1=1	1=1	2=2	2=1+1
3=3	3=2+1	4=!+3	4=2+1+1
5=3+2	5=1+1+3	6=1+3+2	6=3+2+1

We can see that each "circular sum" from 1 to 6 occurs exactly twice (*R*=2). We say that this 1-D GNR has the parameters *n*=4, *R*=2.

The simplest combinatorial configurations, namely Gold Numerical Rings (GNR)s are cyclic sequences of integers which form perfect partitions of a finite interval [1,s] of integers. The sums of connected sub-sequences of an GNR enumerate the set of integers [1,s] exactly *R*-times.

3. 2-D PERFECT ORDERED RING COMBINATORIAL CONSTRUCTIONS

If we regard the ring-ordered *n*-stage sequence $C^{2\text{-}D}_n$={$(k_{11}, k_{12}), (k_{21}, k_{22}), ...,(k_{n1}, k_{n2})$} of 2-D terms as being cyclic, so that (k_{n1}, k_{n2}) is followed by (k_{11}, k_{12}), we call this an 2-D ring sequence. A sum of consecutive terms in the 2-D ring sequence can have any of the *n* terms as its starting point, and can be of any length.

Let us consider 2-D ring sequence of the four (*n*=4) terms $C^{2\text{-}D}_n$= {(1,1), (1,2), (1,4), (1,3)} (Fig.4).

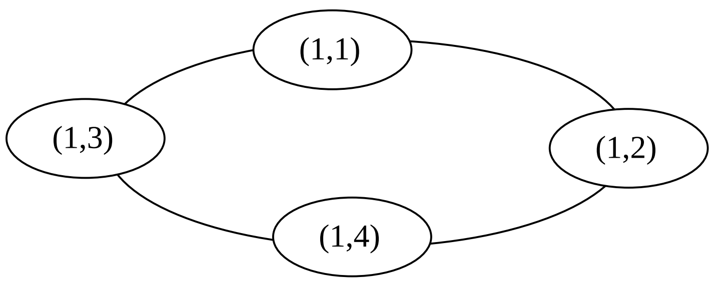

Fig.4 2-D ring sequence of the four (*n*=4) terms {(1,1), (1,2), (1,4), (1,3)}

Using circular 2-D sums, it is easy to calculate all the sums, taking modulo m_1=4 for the first component of the 2-D sums and modulo m_2=5 for the second component of the sums (Table 4).

Tab.3 2-D sums of consecutive terms of the 2-D ring sequence {(1,1), (1,2), (1,4), (1,3)} 2-modulo {4, 5}

(1,1)= (1,1)	(1,2)= (1,2)	(1,3)= (1,3)	(1,4)=(1,4)
(2,1)≡(1,2)+(1,4)	(2,2)≡(1,4)+(1,3)	(2,3)≡(1,1)+(1,2)	(2,4)≡(1,3)+(1,1)
(3,1)≡(1,3)+(1,1)+(1,2)	(3,2)≡(1,1)+(1,2)+(1,4)	(3,3)≡(1,4)+(1,3)+(1,1)	(3,4)≡(1,2)+(1,4)+(1,3)

So long as the elements of the Gold Numerical Ring themselves are circular 2-D sums too, the circular 2-D sums set will be as follows:

$$\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \end{matrix} \qquad (4)$$

The result of the calculation forms the 3×4 - matrix, which exhausts the circular 2-D sums and each of its meets exactly once (R=1). So, the ring sequence of the 2-D vectors {(1,1),(1,2),(1,4),(1,3)} is 2-D Gold Numerical Ring with parameters n=4, R=!, and m_1 =4, m_2 =5 (Fig.4).

In an underlying manner it is possible to consider 3-D as well as multidimensional "gold" numerical combinatorial constructions with numerous terms, and each of the terms can be of any length.

4. APPLICATION OF 2-D GNRS FOR PLANAR LOW-SIDE LOBE ANTENNA DESIGN

Here is example of constructing the planar antenna array configuration, based on two-dimensional GNR with parameters n=13, R=4, m_1 =5, m_2 =8, where $S = m_1 \times m_2$= 5 × 8 = 40, which allows be reconstructed into antenna array over 5 × 6 =30 grids:

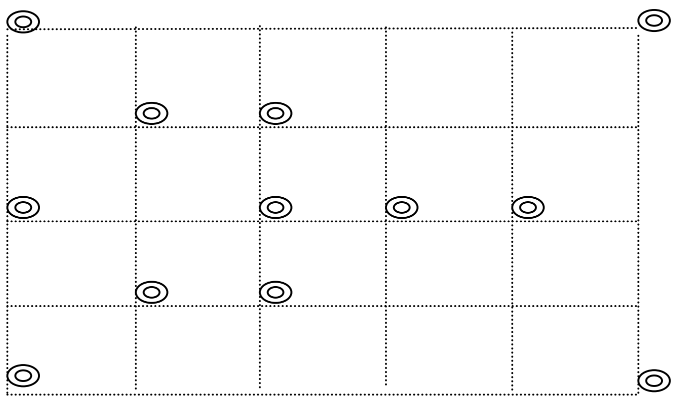

Fig.5 Antenna array over 5 × 6 grids reconstructed from the array over 5 × 8 grids based on the 2-D GNR.

We search needed solution after construction of 2-D matrix of all circular two-dimensional vector-sums on the Gold Numerical Ring and regarding each of its with respect to search minimum of the sum using crossing out. The method described is given Fig. 5.

Underlying procedures make it possible to configure antenna arrays with the smallest possible number of grids.

The example shows that the grid order based on the combinatorial configuration can be reduced further without loss of the possibility to construct an antenna array. For any two-dimensional phased antenna array configurations the antenna or sensor elements are positioned in a manner as prescribed by appropriate 2-D Gold Numerical Ring.

The present method relates to constructing planar phased antenna array configurations. The antenna or sensor elements positioned in a manner as prescribed by underlying combinatorial technique make it possible to configure systems, using appropriate variant of 2-D Gold Numerical Ring for constructing radar or sonar planar antenna arrays with minimum side-lobes. The method involves technique for minimizing sizes of the arrays prescribed by parameters of appropriate 2-D Gold Numerical Ring.

The search algorithm allows finding optimal solution in the simplest way based on regarding the appropriate matrix of circular two-dimensional vector-sums on suitable 2-D Gold Numerical Ring as well as crossing out operations. These procedures make it possible to configure planar antenna arrays with the smallest possible number of grids. Clearly we keep known relationships of grids sizes and parameters of working range for configured radar or sonar system.

5. APPLICATION OF 3-D GNRS FOR CONFIGURE OF HIGH PERFORMANCE SONAR SYSTEMS

Three-dimensional Gold Numerical Rings (3-D GNRs) are ring *n*-stage sequences of 3-stage ordered sub-sequences of *n* integer 3-D vectors, which form "perfect" 3-D partitions of a finite 3-D "circular"-space interval of the vectors over $m_1 \times m_2 \times m_3$. Needed solution is construction of 3-D matrix, based on appropriate 3-D GNR, for which set of all circular 3-D sums of the consecutive terms 3-modulo m_i, i=1,2,3 enumerate a set of 3-stage terms exactly *R*-times. It is a ring sequence $C^{3\text{-}D}_n=\{(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}), \ldots, (k_{n1}, k_{n2}, k_{n3})\}$ of 3-D terms as being cyclic, so that (k_{n1}, k_{n2}, k_{n3}) is followed by (k_{11}, k_{12}, k_{13}). A sum of consecutive terms in the 3-D ring sequence can have any of the *n* terms as its starting point, and can be of any length. However, we require to keep condition:

$$(m_1, m_2, m_3)=1 \qquad (5)$$

Such model make it possible to configure 3-D grid over $m_1 \times m_2 \times m_3$, each node of the grid meets exactly *R*-times.

For example, the ring ordered 6-stage (*n*=6) sequence of 3-stage terms {(0,1,4), (0,2,4), (1,1,1), (1,1,2), (1,0,3), (0,2,2)} allows form 3-D matrix over 2 × 3 × 5 grid, based on the 3-D GNR. Here all circular 3-D vector sums, taking 3-modulos m_1=2, m_2=3, m_3=5 enumerate nodes of the grid exactly once (R=1):

(0,0,1) ≡ (0,2,2) + (0,1,4)
(0,0,2) ≡ (1,1,2) + (1,0,3)
(0,0,3) ≡ (0,1,4) + (0,2,4)
(0,0,4) ≡ (1,0,3) + (0,2,4) + (0,1,4) + (0,2,4) + (1,1,1)
....
(0,0,0) ≡ (0,1,4) + (0,2,4) + (1,1,1) + (1,1,2) + (1,0,3) + (0,2,2)

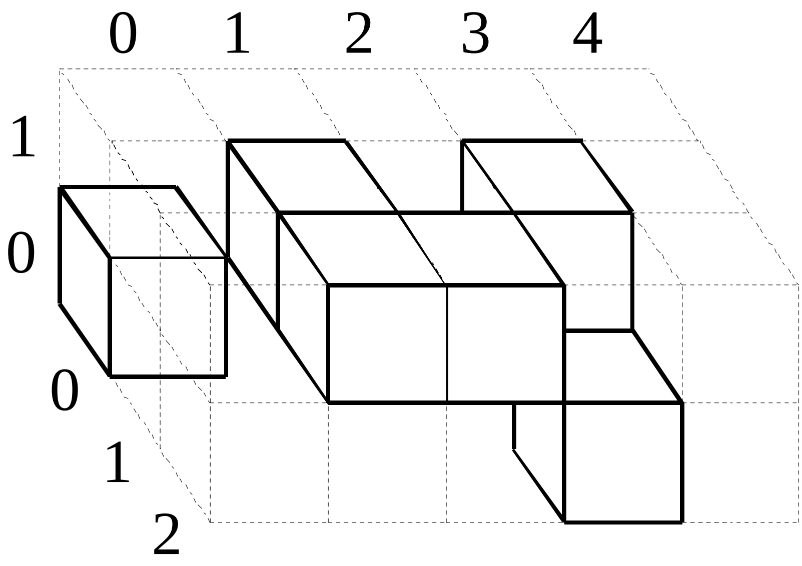

Fig.6 Geometric structure of the 3-D sonar system, based on the 3-D GNR. {(0,1,4), (0,2,4), (1,1,1), (1,1,2), (1.1,3), (0,1,2)}

Now, we can obtain coordinates of remaining five elements (1,1,1), (1,2,2), (1,1,3), (1,2,1), (0,2,3) 3-modulos (2,3,5) accordingly to the underlying 3-D Gold Numerical Ring, which configure 3-D sonar system. After this procedures we configure geometric structure of the 3-D sonar system, based on the 3-D GNR. {(0,1,4), (0,2,4), (1,1,1), (1,1,2), (1.1,3), (0,1,2)} over $2 \times 3 \times 5$ - grids (Fig.6).

It should be noted that after crossing out operations on the $2 \times 3 \times 5$ grids we can have 3-D antenna array over $2 \times 3 \times 4$, due to exclude all right-hand columns.

The example shows that the grid order based on the three-dimensional Gold Numerical Rings can be reduced further without loss of the possibility to construct an antenna array. For any three-dimensional phased antenna array configurations the antenna or sensor elements positioned in a manner as prescribed by appropriate 3-D Gold Numerical Ring.

6. APPLICATION OF MULTIDIMENSIONAL GNRS FOR VECTOR DATA CODING

A more general type of the "perfect" combinatorial configurations are *t*-dimensional (t>1) Gold Numerical Rings, where *t*-D GNR can be represented as an *n*-stage cyclic sequence $\{(k_{11}, k_{21}, \dots k_{t1}), (k_{12}, k_{22}, ,\dots k_{t2}), \dots (k_{1n}, k_{2n}, \dots k_{tn})\}$.

The *t*-D GNR allows configure a set of circular *t*-vector-sums on the sequence over $M_1 \times M_2 \times \dots \times M_t$ -matrix exactly *R*- times.

Graphical model of *t*-D GNR is given below (Fig.7).

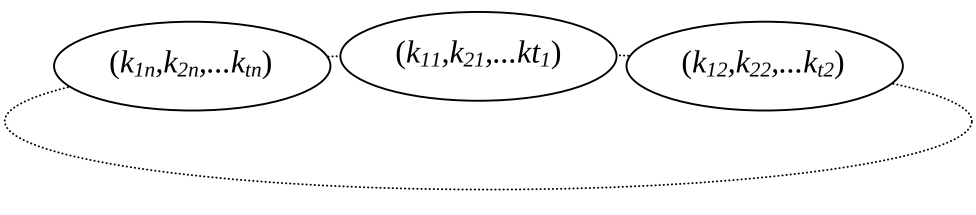

Fig.7 Graphical model of *t*-D GNR

Traditional methods for vector data coding are not always applicable because optimization technique that could be used to solve such problem demands to revise a lot of options. To solve the problem the property of *t*-dimensional GNR can be put into the basis of design the *t*-dimensional Monolithic GNR-code (MGNR-code). The code forms code combinations of semi-type symbols, which follow each other.

For example, the ring-like 2-D MBC formed on two-dimensional (*t*=2) GRB {(1,1),(1,2),(1,4),(1,3)} is illustrated by the next table:

1000	0100	0001	0010
0110	0011	1100	1001
1101	1110	1011	0111

The table consists of all code combinations by 2-D MGNR-code of 2-vectors from (1,1) to (3,4) the only way.

Applications profiting from the code are for example compression, signal reconstruction, operation speed , and simplicity of coding systems.

The underlying multidimensional models make it possible to apply concept of the Gold Numerical Rings for configure high-performance vector data communication systems, based on combinatorial techniques, with direct applications to information coding and modulation.

7. CONCLUSION

The Perfect Combinatorial models discovers, essentially, a remarkable property of a space, and provides the new mathematical principles relating to the optimal placement of structural elements in spatially or temporally distributed systems. This property makes Gold Numerical Rings useful for finding optimal placement of structural elements in spatially distributed sonar or acoustic systems. Applications profiting from Gold Numerical Rings theory are resolving ability, minimizing sizes, non-redundant aperture and low-side lobe antenna arrays, vector data coding and signal reconstruction. There are a lot of multidimensional Gold Numerical Rings, which can be well used for configure high performance acoustic or sonar systems.

The perfect numerical models are based on the idea of perfection and harmony, which exists not only in the abstract models but in the real spice-time also.

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