# GOLAY'S CODES SEQUENCES IN ULTRASONOGRAPHY

ANDRZEJ NOWICKI, IGOR TROTS, WOJCIECH SECOMSKI, JERZY LITNIEWSKI

Department of Ultrasound, Institute of Fundamental Technological Research,
Polish Academy of Sciences
Warsaw, Świętokrzyska 21, Poland
anowicki@ippt.gov.pl

*The issue of maximizing penetration depth with concurrent retaining or enhancement of image resolution constitutes one of the time invariant challenges in ultrasound imaging. Concerns about potential and undesirable side effects set limits on the possibility of overcoming the frequency dependent attenuation effects by increasing peak acoustic amplitudes of the waves probing the tissue. To overcome this limitation a pulse compression technique employing 16 bits Complementary Golay Code (CGS) was implemented at 4 MHz. In comparison with other, earlier proposed, coded excitation schemes, such as chirp, pseudo-random chirp and Barker codes, the CGS allowed virtually side lobe free operation. Computer simulation results for CGS pulse compression are presented. Next three different methods and algorithms used to calculate the pairs of Golay sequences of the different length are described. Experimental results are presented in the form, which in clear way illustrates the resolution, signal penetration and contrast dynamics of ultrasonic images obtained by using Golay coded excitation.*

## INTRODUCTION

Maximum range / tissue penetration - and fine range resolution which are the two most important considerations in ultrasonographic imaging are contradicting demands.

Sound absorption in the tissue increases approximately linearly with frequency thus limiting the resolution in investigating of the deep structures. On the other hand the possible biological effects related to the insonification limit the probing peak power. This limitation can be overcome by using long wide band transmitting sequences and compression techniques on the receiver side

[9]. To this end different processing systems were proposed in Non Destructive Testing (NDT), long range underwater acoustics and recently in medical imaging. Basically all use coded transmitted signals and employ correlation and averaging on reception of the echoes. Consequently the high peak transmitted power is no more required – the gain in SNR results from the compression of the echoes. Extensive comparison of the standard radio frequency (RF) sine wave bursts compared to the random noise transmission and the subsequent compression using polarity coincidence correlator was done by Bilgutay et al [1]. They showed the SNR ratio enhancement especially when integration time of the correlator was made arbitrarily long. However the requirements of the real time medical scanning do not permit such extensive integration time and the attained SNR final gain depends on the length of the transmitted sequence and the efficacy of the compression algorithm.

Among excitation sequences proposed in ultrasonography, Golay codes evoke more and more interest in comparison with other signals. The reason of that lies in the fact that Golay codes, like no other signals, suppress to zero the amplitude of side-lobes. First, these types of complementary sequences have been introduced by Golay [4]. The pairs of Golay codes belong to a bigger family of signals, which consist of two binary sequences of the same length $\boldsymbol{n}$ whose auto-correlation functions have side-lobes equal in magnitude but opposite in sign. The sum of these auto-correlation functions gives a single auto-correlation function with the peak of $2n$ and zero elsewhere.

One of the main problems associated with all the binary codes discussed so far in literature is the high side-lobe level for short code lengths. Since a delta ambiguity function is rather utopian, one solution to the problem can be the utilization of a set of waveforms with complex ambiguity functions, which are in some sense as "different" as possible. By coherently combining the matched-filter responses (the complex ambiguity functions of the set), the restrictions imposed on a single ambiguity function can be bypassed.

Several authors addressed similar boundary-condition problem of signal compression in medical diagnostic imaging. Cohen [2] analysed the principles of pulse compressions in radar system concentrating on the behaviour of the linear frequency modulation and binary phase modulation, using Barker, pseudorandom and Golay codes. Suppression of the side lobes after pulse compression was also extensively examined and it was pointed out that using the pulse compression leads to: a) improvement of the detection performance for a given peak power; b) mutual interference reduction; c) increase in system operational flexibility.

The improvement of the SNR in medical ultrasonic imaging was clearly demonstrated by Haider and al [5]. The SNR gain was achieved by elongation of the excitation pulse and employment of the deconvolution filter which was implemented as a modified Wiener filter with the deconvolution kernel being the excitation waveform. Authors concentrated on the analysis of the Barker codes of length 7 and 13 and pseudochirp signals. Also O'Donnell [8] and Misaridis et al [7] showed the penetration improvement in real-time imaging system when using the coded excitation.

This paper is organized as following. RF transmitted ultrasonic signals with phase modulated according to the Barker and Golay binary codes are theoretically simulated. Next three different methods and algorithms used to calculate the pairs of Golay sequences of the different length are described. Experimental results comparing the echoes from the tissue phantom for burst two period transmission and Complementary Golay Codes in clear way illustrates the gain in resolution, signal penetration and contrast dynamics of ultrasonic images when using Golay coded excitation.

## 1. SYNTHESIS OF COMPLEMENTARY GOLAY SEQUENCES

The most widely used binary codes are the Barker sequences. They are optimum in the sense that the autocorrelation function peak is ***N*** and the side lobe level falls between +1 and-1, where ***N*** is the number of sub pulses (elements). The compression ratio is proportional to the length of the code, however no Barker code larger than 13 elements has been found. For code length 13 "+ + + + + - - + + - + - +" the Peak Side Lobe level *PSL*=10 log (maximum side lobe power / peak response power) is equal to –22.3 dB and Integrated Side Lobe level *ISL*=10 log (total power in the side lobes / peak response power) is equal to –11.5 dB. In this work "+" refers to a pulse amplitude of "1" or positive phase and "-" designates a pulse amplitude of "-1" or negative phase.

In general the range of side lobes level decreases with code length and much longer code length >1000 is required for the 60 dB dynamic range of the ultrasonographic images. Barker codes longer than 13 are not known, however there are combined Barker Codes where much larger pulse compression ratio is achievable [2].

Golay complementary sequences are pairs of binary codes, belonging to a bigger family of signals called complementary pairs, which consist of two codes of the same length *N* whose auto-correlation functions have side-lobes equal in magnitude but opposite in sign. Summing them up results in a composite auto-correlation function with a peak of 2*N* and zero side-lobes. Fig. 1 illustrates the principle of the side-lobe-canceling for a pair of signed of length equal to 8 bits each.

There are essentially several algorithms for generating Golay pairs.

Let the variables $a_i$ and $b_i$ ($i$=1,2,…$n$) are the elements of two $n$-long complementary series equal either '+1'or '-1', [3],

$$A = a_1, a_2, ..., a_n; \qquad (1)$$
$$B = b_1, b_2, ..., b_n.$$

The ordered pair ($A$;$B$) are Golay sequences of length $n$ if and only if their associated polynomials

$$A(x) = a_1 + a_2 x + ... + a_n x^{n-1}, \qquad (2)$$
$$B(x) = b_1 + b_2 x + ... + b_n x^{n-1},$$

satisfy the identity

$$A(x)A(x^{-1}) + B(x)B(x^{-1}) = 2n \qquad (3)$$

in the Laurent polynomial ring $Z[x, x^{-1}]$.

Let their respectable auto-correlation functions $N_A$ and $\boldsymbol{N_B}$ of the sequences $A$ and $B$ respectively be defined by

$$N_A(j) = \sum_{i \in Z} a_i a_{i+j} \qquad (4)$$
$$N_B(j) = \sum_{i \in Z} b_i b_{i+j}$$

where we set $a_k = 0$ if $k \notin (1...n)$. Now condition (3) can be expressed by the sum $N_A + N_B$, and

$$N_A(j) + N_B(j) = \begin{cases} 2N, & j = 0 \\ 0, & j \neq 0 \end{cases} \quad (5)$$

The sum of both autocorrelation function is at j=0 and zeroing otherwise.

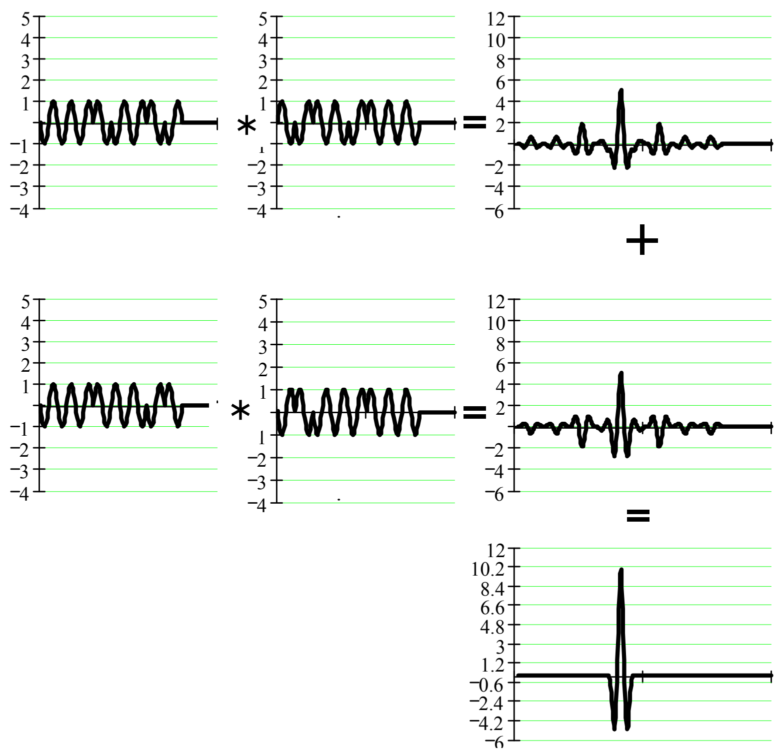

Fig. 1. Principle of side lobe cancellation using pair of Golay complementary sequences of length 8

The second, recursive method for constructing the Golay's sequences is presented below.

Let the variables *a*(i) and *b*(i) be the elements (*i*=0,1,2,… $2^n$ -1) of two complementary sequences with elements +1 and –1 of length $2^n$

$$\begin{aligned} a_0(i) &= \delta(i) \\ b_0(i) &= \delta(i) \end{aligned} \quad (6)$$

$$\begin{aligned} a_n(i) &= a_{n-1}(i) + b_{n-1}(i - 2^{n-1}) \\ b_n(i) &= a_{n-1}(i) - b_{n-1}(i - 2^{n-1}) \end{aligned} \quad (7)$$

where $\delta(i)$ is the Kronecker delta function.

Equation (7) shows that in each step the new elements of the sequences are produced by concatenation of elements $a_n(i)$ and $b_n(i)$ of the length $\boldsymbol{n}$.

*Example:*

Let $n$=1, than $i$ takes values 0 and 1.

$$a_1(0) = a_0(0) + b_0(-1) = 1;$$
$$b_1(0) = a_0(0) - b_0(-1) = 1;$$
$$a_1(1) = a_0(1) + b_0(0) = 1;$$
$$b_1(1) = a_0(1) - b_0(0) = -1.$$

As final results we obtain two complementary sequences of the length $2^n$:

$a_1$={1, 1};

$b_1$={1, -1}.

Once these operations are performed recursively for $\boldsymbol{n}$=2,3,4… the following complementary sequences are obtained:

$a_2$={1, 1, 1, -1};

$b_2$={1, 1, -1, 1}.

$a_3$={1, 1, 1, -1, 1, 1, -1, 1};

$b_3$={1, 1, 1, -1, -1, -1, 1, -1}.

$a_4$={1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1};

$b_4$={1, 1, 1, -1, 1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, 1}.

Similar method of generating the complementary code pairs, differing only in the applied mathematical formalism has been described by Mendieta and al [6]. This method can be applied to sequences of length $n$ to obtain another code pair of length $2n$

$$\begin{bmatrix} \dfrac{A}{B} \end{bmatrix} \rightarrow \begin{bmatrix} \dfrac{A \oplus B}{A \oplus \overline{B}} \end{bmatrix} \tag{8}$$

where $\overline{B}$ is the inverse of $B$ and $\oplus$ indicates concatenation of functions. This procedure may be iterated in the following way

$$\begin{bmatrix} \dfrac{A}{B} \end{bmatrix} \rightarrow \begin{bmatrix} \dfrac{A \oplus B}{A \oplus \overline{B}} \end{bmatrix} \rightarrow \begin{bmatrix} \dfrac{(A \oplus B) \oplus (A \oplus \overline{B})}{(A \oplus B) \oplus (\overline{A \oplus \overline{B}})} \end{bmatrix} \rightarrow \begin{bmatrix} \dfrac{((A \oplus B) \oplus (A \oplus \overline{B})) \oplus ((A \oplus B) \oplus (\overline{A \oplus \overline{B}}))}{((A \oplus B) \oplus (A \oplus \overline{B})) \oplus \overline{((A \oplus B) \oplus (\overline{A \oplus \overline{B}}))}} \end{bmatrix}$$

For example, starting with the one element Golay pair, the Golay codes of length 2,4,etc. are derived:

$$\left[\frac{1}{1}\right] \to \left[\frac{1 \quad 1}{1 \quad -1}\right] \to \left[\frac{1 \quad 1 \quad 1 \quad -1}{1 \quad 1 \quad -1 \quad 1}\right] \to \left[\frac{1 \quad 1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1}{1 \quad 1 \quad +1 \quad -1 \quad -1 \quad -1 \quad 1 \quad -1}\right] \to etc.$$

Although these codes may seem to represent the ideal solution to the side lobe suppression problem, but in practice tissue and especially blood is moving between the two transmits, so perfect cancellation between the two firings will not be achieved.

## 2.EXPERIMENTAL RESULTS

The aim of this experiment was to investigate the special features of Golay codes behaviour, their advantages in comparison with ordinary sine-like signal and Barker phased modulated sine sequences. The block diagram of the experimental setup is shown in the Fig. 2.

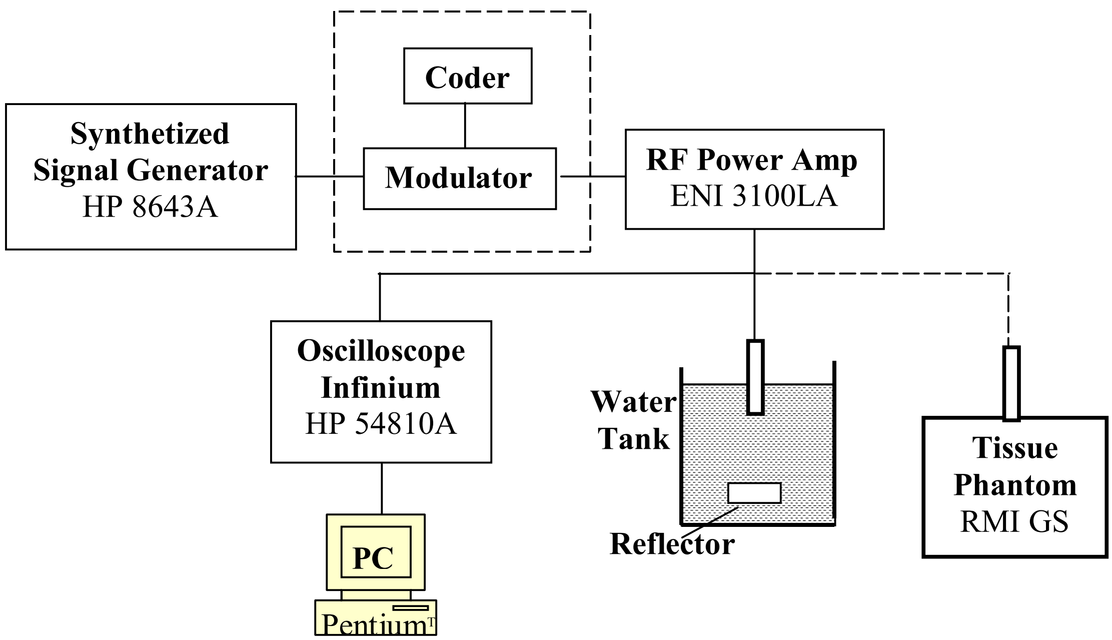


Fig. 2. The experimental setup

The sinusoidal signals at the frequencies of 4 MHz were synthesized using Signal Synthesiser (HP8643A, Agilent, USA). This signal was connected to the bipolar modulator driven by the {0,1} sequences from the custom design coder. The coder preludes programmed logic (EPM7064, Altera$^{TM}$, USA) allowed to generate one of different transmitter functions: {1,1} sequence resulting in 2 periods of the sine wave and switched pair of 8 or 16 bits Golay codes as well as 7 and 13 bits Barker codes. The PRF of the transmitted signals was set to 1 kHz. After amplification in the power RF amplifier (ENI 3100LA, USA) the transmitter burst were exciting the ultrasonic transducer immerged in water tank or moved over the Tissue phantom (GS, RMI, USA). The excitation voltage applied to the ultrasonic transducer was equal to 50 $V_{p-p}$ for all different transmitted sequences in order to keep the $I_{SPTP}$ intensity constant (Mechanical Index constant). The coder consists of comparator, PLD logic and analog multiplier. The input signal, sine wave at 0dBm level, is multiplied either by "1" (output in-phase) or "-1" (output out-of-phase) or "0" (no output signal). The PLD logic is a divide-by-N counter, generating cyclic sequences with pulse repetition frequency and sequence ROM, where different codes are stored.

The first part of the experiment was to compare the effective overall axial resolution for both transmission modes, namely sine burst and CGS.

Rectangular shape, 2 mm thick perplex sheet acting as a reflector was mounted in the water tank at the axial distance equal to 6 cm. This distance corresponds to the focal distance of the 3.5 MHz transducer used for the experiments.

The RF echoes data were acquired using digital oscilloscope, with a sampling rate 40 ns (25 MHz). Next, the collected digital data were processed off-line and displayed on the oscilloscope. The processing included amplification, pulse compression for Golay sequences, and envelope detection.

Fig.3 shows the RF echoes for two periods sine burst transmission (top), and time compressed complementary Golay series (bottom), respectively.

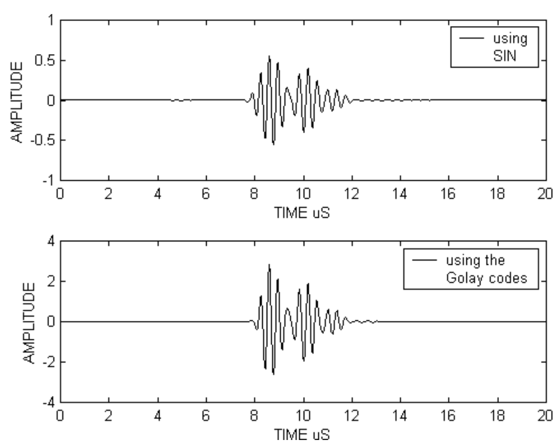


Fig. 3. Echoes from the reflector - plastic sheet, thickness 2 mm. Two periods sine burst (top) and Golay codes (bottom)

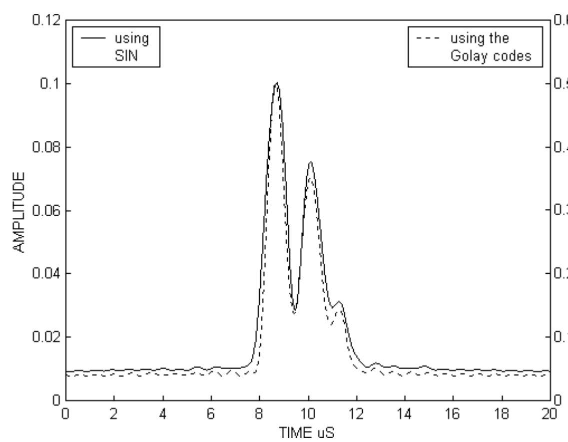


Fig. 4. Comparison of results with using sine-like signal and sequences of Golay codes - envelopes.

It can be seen that both echoes from the front and rear surface of the perplex reflector are basically identical in shape, however the amplitude of the compressed echoes is approximately 5 times larger than that received from the sine burst transmission. This value is close to the theoretically predicted gain for 8 bits CGS compared to that achievable with the 2 cycles sine burst. Ideally, the expected gain should be 8, however, the finite bandwidth of the pulse-echo transducer slightly elongates the pulse durations and lowers the effective gain.

In Fig.4 the envelopes of the detected echoes shown in Fig.3 are depicted. To facilitate the comparison the amplitudes of the envelopes were normalized and it can be seen that their shape is virtually identical. That indicates that rather elaborated reception/compression algorithm for Golay series does not modify or disturb the received signal (here sine burst echoes are considered to be the reference ones).

The next set of experiments was carried out using tissue phantom (GS RMI, Wisconsin, USA) and the results obtained are shown in Fig. 5.

Two images of a tissue phantom with attenuation of 0.7 dB/[MHz $\times$ cm] are shown in Fig.5. It consists of the nylon wires of 0.374mm in diameter, positioned every 1 cm axially. Additional wires are placed at a 30 degree angle at the top of the phantom. Also some wires are placed in depth 3 cm with decreasing distance down from 3mm to 0.5mm.

The two-cycle pulse of the frequency 4 MHz and the pair of the Golay codes of the length 16 bits and at the same frequency were used. The peak pressure levels of the excitation signals at the transducer were set as low as possible to visually detect the echoes received using burst transmission slightly larger than the noise level. The same peak pressure was used for coded transmission. The scanned area of the phantom is mark by rectangle (Fig. 5c).

The resulting images are shown in Fig.5a and Fig.5b. For quantitative comparisons, the RF-lines are also shown.

The SNR gain when moving from burst to coded transmission is evident. Applying conventional pulses results in penetration reaching hardly 4 cm. The scan distance obtained with Golay coded transmission extends down to 7 cm (lowest visible white dot at the image), Fig.5b. The respective RF-echo lines shown below each image the central RF-lines are plotted confirm the outstanding quality of the signal received, when the Golay coded transmission was used.

These two images clearly demonstrate that abdominal ultrasound imaging can benefit from Golay sequences yielding a higher SNR and therefore deeper penetration, while maintaining both axial and lateral resolution. The range resolution that can be achieved is always higher to that of a conventional system. However, confirming our hypothesis, the cancellation of the side-lobes is not perfect due to attenuation, and shadows are still visible along the wires. The main disadvantage of Golay pairs is that it requires two transmitting events for every line that decreases the frame rate by half. Also, they are sensitive to motion. In practice, tissue and especially blood is moving between the two transmits, so perfect cancellation between the two firings will not be achieved.

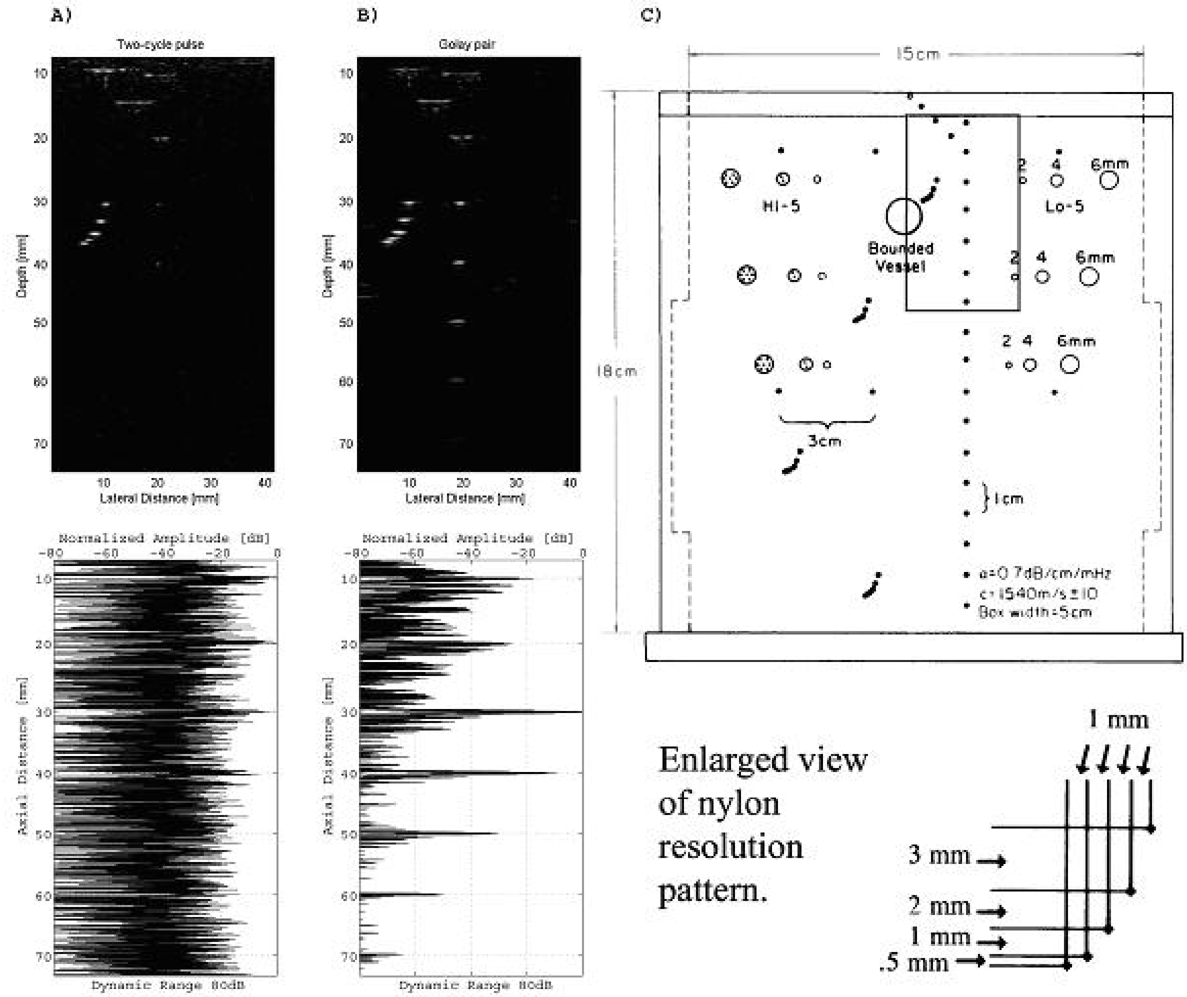


Fig. 5. Ultrasonic images of the RMI 415 GX tissue phantom obtained using conventional two cycle sine burst transmission (a) and 16 bit Golay coded transmission (b). Below each image the central RF-lines recorded using the respective transmission are plotted. c) Schematic diagram of the tissue phantom under examination

## 3. CONCLUSION

Some methods for calculating the pairs of Golay sequences of different lengths, which can be used in ultrasonography were described. Transmission of long coded sequences and compression of the received echoes by means of the matched filtering allow to obtain axial resolution better to that obtained using burst transmission but with considerably higher amplitude. Using Golay sequences allow improving the SNR that plays the main role in ultrasonographic imaging. For example, SNR is equal to 24.1dB when Golay sequences were used and 8.4dB for sin burst excitation at the depth of 30mm, improving the contrast resolution. This makes it possible to explore the signals with lower amplitude that in its turn is very important since it decreases the patients' exposure to ultrasound. Another important reason of using Golay sequences is the fact that they allow using of higher frequencies, improving the imaging resolution. The experimental studies of reconstructed images showed over twice deeper penetration without sacrificing the axial resolution and contrast resolution. Even the shallow structures are more clearly visualized with improved contrast resolution, thanks to removal off the spurious clutter reflections.

The improved quality of the CGS image is noticeable. As discussed previously, this image exhibits the gain of 9.6 dB in signal-to-noise ratio in comparison to that produced by the sine burst transmission. The noise present in the sine burst image is clearly suppressed in the CGS one, indicating considerable improvement in dynamic contrast.

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