

## Introduction to tissue shear wave elastography

Andrzej NOWICKI, Katarzyna DOBRUCH-SOBCZAK

Institute of Fundamental Technological Research, Polish Academy of Sciences  
Pawinskiego 5B, 02-106 Warsaw, Poland  
anowicki@ippt.gov.pl

*Ultrasonic elastography is a technique allowing imaging of the elastic properties of tissue. There are two basic techniques of elastographic imaging; compressional - displaying the evaluation of tissue deformation under the external stress; and dynamic, tracking the propagation velocity of the shear wave generated by the acoustic radiation force. Soft tissue bulk modulus varies, from a few to several GPa, whereas the shear modulus is significantly smaller, not exceeding a few hundred Pa for adipose tissue, breast or liver, up to several hundred kPa for "hard" tissue. Forces generated in the tissue due to the external, axial piston-like stresses depend mainly on the shear modulus. In Shear Wave Elastography, long, several tens of microseconds, ultrasonic pulses successively focused at several depths are sent: generating a conical wave front moving with the "supersonic" velocity, depending on the tissue stiffness. Velocity of propagation of shear wave depends on the shear modulus  $\mu$  and the modulus of elasticity  $E$  of the examined tissue is equal to  $E=3\mu$ .*

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### 1. Introduction

Palpation of superficial organs and abdominal cavity, chest, and auscultation of patients with a stethoscope are standard diagnostic procedures. During palpation, a physician applies pressure to the body of the patient, sensing the position, hardness, mobility and position, of the examined organs.

In ancient Egypt palpation was one of the basic forms of physical examination. Quoting from the Ebers papyrus from about 1550 BC, "...sensing palpation of the heartbeat is very important, also palpation of the stomach - if you examine a person suffering from visceral troubles and under the influence of pressure the „lesion" runs away from the fingers like oil in a leather bag, then put him on his back. If the stomach is warm and hard then it is the liver problem - prepare special, secret herbs. " Also, in traditional Chinese medicine, touching the body to sense the pulse in the radial artery, and from the nature of the pulsation to assess the state of the disease, belonged to the sophisticated medical arts, and first reference comes from the period 500 BC, from the days when Bian Que lived.

Between 2009 and 2010, 22% of the 38 million imaging procedures in the National Health Service in England were performed using ultrasonography; this accounted for nearly 30% more procedures than computer tomography, magnetic resonance and nuclear medicine combined [9]. The differences between imaging techniques result mainly from the physical mechanism that produces image contrast. In Rtg the contrast depends mainly on the differences in the atomic number of the imaging material, the RMI on the proton distribution, and the differences in their relaxation times. Ultrasound contrast depends on the variations in acoustic impedance of organs – product of density and longitudinal velocity - the latter depends on the elastic modulus.

Tissue deformation is only a certain approximation of the stiffness of the tissue that the physician feels during palpation: small tissue deformation corresponds to high stiffness and vice versa. For organs with simple geometry, if we know the stress and strain then their ratio is the modulus of elasticity. The elasticity of the material describes its ability to return to the initial shape, after the material has been deformed by external stress. For a homogeneous isotropic solid, the stress/strain ratio is a constant, and is called the modulus of elasticity, or Young's modulus  $E$ .

Liquids resist the change in volume, and not in shape, and that is why they only have volume elasticity. Solids resist changes in shape and in volume, so they possess shear elasticity.

Biological tissues are almost incompressible, and their Poisson's ratio  $\gamma$  is in the range 0.490 - 0.499, which means that the tissues are almost incompressible (for an incompressible liquid Poisson's ratio  $\gamma = 0.5$ ).

Bulk modulus  $B$  describes the change in the material's volume under external stress, and is equal to

$$B = \frac{E}{3(1-2\gamma)} \quad (1)$$

Bulk modulus  $B$  of most soft tissues differs not more than 15% from the B modulus for water. The high water content in biological tissues means that it is easy to change their shape after compression, but the volume is preserved.

The shear modulus  $\mu$  changes over a large range, several orders of magnitude. Thus, shear and Young's moduli that have the widest range of dynamics are the most appropriate parameters for assessing tissue stiffness and are the closest to what is felt in palpation.

Modulus of elasticity and shear modulus of tissue are linked by a simple scale ratio that equals 3,  $E=3\mu$ . Tissue deformation is only a certain approximation of the tissue stiffness, that a doctor feels during palpation: small tissue deformation corresponds to high stiffness and vice versa. Thus, in practice, the deformation brings important information about the state of the tissue. For organs with simple geometry, if we know the strain and deformation, their ratio is the modulus of elasticity.

Because tissue is, in the general case, a solid matrix immersed in liquid, its properties lie between the two materials. The shear modulus  $\mu$  is so small that the axial stress induces a very large change in lateral dimensions of the material, and hence its "structure". As a result, the ultrasonic echoes returning from inside of the material before, and after, the compression are different.

In the temperature range of 20°C - 40°C the compressibility factor varies from about  $4.6 \cdot 10^{-10} \text{Pa}^{-1}$  to  $4.1 \cdot 10^{-10} \text{Pa}^{-1}$  [10].

The  $\mu$  ratio to the solids modulus does not exceed several tenths, for most materials from 0.2 to 0.4. For liquid this ratio is equal to zero. Liquids are incompressible and Poisson's constant is 0.5. Poisson's coefficients in tissue tissues range from about 0.49 to 0.499.

Sarvazyan [7] has shown that the forces produced in the tissue under axial compression of the piston depend mainly on the shear modulus. For a piston with radius  $r$  and displacement  $\Delta l$ , this force is

$$F = \frac{8\mu r \Delta l}{1 + \frac{\mu/B}{1 + \mu/3B}} \quad (2)$$

Because  $\mu \ll B$ , then the second component in the denominator disappears, and the relationship between the displacement of the piston and the force depends on the shear module. This means that palpation and elastography do not depend on the compressibility modulus.

## 2. Acoustic radiation force

The propagation of the pressure wave is accompanied by two phenomena - radiation pressure and mass flow, called streaming, [5, 6]. Both of these phenomena will be discussed using Euler's power equation and the principle of conservation of mass. For both equations we insert the acceleration of the particle, taking into account its local derivative and convective derivative  $\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial x}$ . For plane wave propagating towards  $x$  direction

$$\frac{\partial p}{\partial x} + \rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} = 0 \quad (3)$$

and

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial x} + v \frac{\partial \rho}{\partial x} = 0 \quad (4)$$

where  $\rho$  is density and  $v$  is particle velocity.

After multiplying (4) by particle velocity  $v$  and adding to (3) we obtain

$$\frac{\partial p}{\partial x} + \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2)}{\partial x} = 0 \quad (5)$$

Integration of (5) with  $x$  leads to

$$p = -\rho v^2 - \int \frac{\partial(\rho v)}{\partial t} dx \quad (6)$$

Time averaged of the expression under the integral disappears, and in the plane wave only the DC component  $\langle -\rho v^2 \rangle = -\frac{1}{2} \rho v^2$  called radiation pressure  $\Pi$  is left.

$$\Pi = -\frac{1}{2} \rho v^2 \quad (7)$$

The intensity  $I$  of the wave in the medium is equal to the amount of energy  $E$  in the cuboid with a unitary base and the height  $c$  (sound speed in the medium).

$$I = E \cdot c = \frac{1}{2} \rho c v^2 \quad (8)$$

By comparing (7) and (8) we obtain the relation between radiation pressure and intensity

$$\Pi = \frac{I}{c} = E \quad (9)$$

The radiation pressure has a component in the direction of the wave propagation, and in contrast to the scalar hydrostatic pressure field, it is a tensor. The acoustic radiation force  $F$  is due to the absorption phenomenon - it acts in the direction of radiation and is equal to the product of radiation pressure and damping

For total absorption we have

$$F = \frac{2\alpha I}{c} \quad (10)$$

where  $\alpha$  is the absorption coefficient.

The spatial distribution of the radiation force field is, thus, determined by both the transmitted acoustic parameters and the tissue properties. When radiation force is applied to a given spatial volume for a short duration, transient shear waves are generated that propagate away from the initial region of excitation, [7]. Radiation power at full absorption is about  $7 \cdot 10^{-4} \text{ N} \cdot \text{W}^{-1}$ . For total absorption of waves of 1 W power propagating in tissue, the radiation force  $F$  is equal to approximately 0.7 mN - a force close to the force of gravity acting on a mass of 70 mg.

The deformation field  $\varepsilon(x, r, \theta)$  resulting from the acoustic radiation force is determined from the analytical solution for point load in elastic medium [4, 8].

$$\begin{aligned} \varepsilon_x &= \frac{-\left(Fx(1+\gamma)(4x^2(-1+\gamma)+r^2(-1+4\gamma))\right)}{8E\pi(r^2+x^2)^{5/2}(-1+\gamma)} \\ \varepsilon_r &= \frac{-\left(Fx(-2r^2+x^2)(1+\gamma)\right)}{8E\pi(r^2+x^2)^{5/2}(-1+\gamma)} \\ \varepsilon_\theta &= \frac{-\left(Fx(1+\gamma)\right)}{8E\pi(r^2+x^2)^{3/2}(-1+\gamma)} \end{aligned} \quad (11)$$

where  $F$  is the amplitude of the radiation force,  $r$  - radial distance from the center of the applied force,  $x$ -distance along the radiation axis,  $E$ -Young modulus and  $\gamma$  is Poisson's constant.

The displacement  $u_x$  of the material is obtained after integration of the strain  $\varepsilon_x$  by the variable  $x$ ,

$$u_x = \frac{(F(1+\gamma)(4x^2(-1+\gamma)+r^2(-3+4\gamma)))}{8E\pi(r^2+x^2)^{3/2}(-1+\gamma)} \quad (12)$$

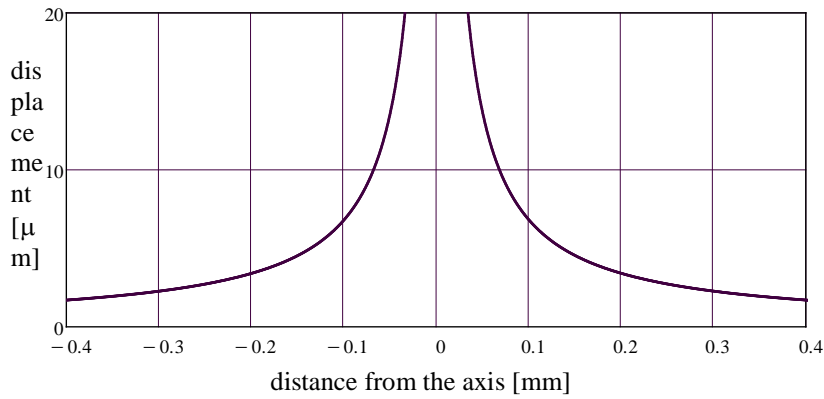


Fig.1. Axial displacement under the influence of acoustic radiation force,  $F=1.7 \cdot 10^{-5} \text{ N}$ ,  $E=3 \cdot 10^3 \text{ Pa}$   $\gamma=0.495$ .

The speed of propagation of the mechanical wave in a solid body is

$$c_l = \left( \frac{B}{\rho} \right)^{\frac{1}{2}} \quad (13)$$

$$c_s = \left( \frac{\mu}{\rho} \right)^{\frac{1}{2}} \quad (14)$$

The speed of longitudinal waves  $c_l$  varies from 1410 m/s in fat, to about 1630 m/s in muscles, the density  $\rho$  varies from  $916 \text{ kg/m}^3$  to  $1060 \text{ kg/m}^3$ , respectively. By substituting these values for the pattern for the modulus  $B$ , it changes from 1800 MPa for adipose tissue, to about 2800 MPa in the muscle. The transverse wave velocity  $c_s$  in the tissues is several hundred times lower than the longitudinal wave velocity, and hence the shear modulus  $\mu$  for different tissues are several orders of magnitude smaller than the corresponding elastic modulus  $B$ .

### 3. Shear wave Elastography

There are two major kind of dynamic Elastography modalities: ARFI Imaging Acoustic Radiation Force Imaging (ARFI) and Shear Wave Elastography (SWE).

In the ARFI (Acoustic Radiation Force Imaging) method, the push pulse deforms the tissue, Fig.2. The size of the deformation is monitored by correlating the successive ultrasonic echoes from the recorded ultrasound images. Both sizes, tissue deformation (spatial displacement derivative) and displacement are inversely proportional to tissue stiffness.

In SWE the transverse wave is produced by strongly focused ultrasound (Acoustic Radiation Force). The speed of the shear wave propagation depends on the stiffness of the tissue. Bercoff et al [1, 2] have described and constructed an ultrasound designed to detect

shear wave propagation phenomena in tissues. By computer modeling, the authors have shown that the radiation-induced shear wave depends on the visco-elastic properties of the tissue.

Shear waves dissipate in tissues much slower, from 1m/s to 10m/s, compared to the velocity of longitudinal waves. Shear waves dissipate well in the elastic media, not in the water, which is rapidly attenuated. Their propagation distance does not exceed a dozen or so millimeters.

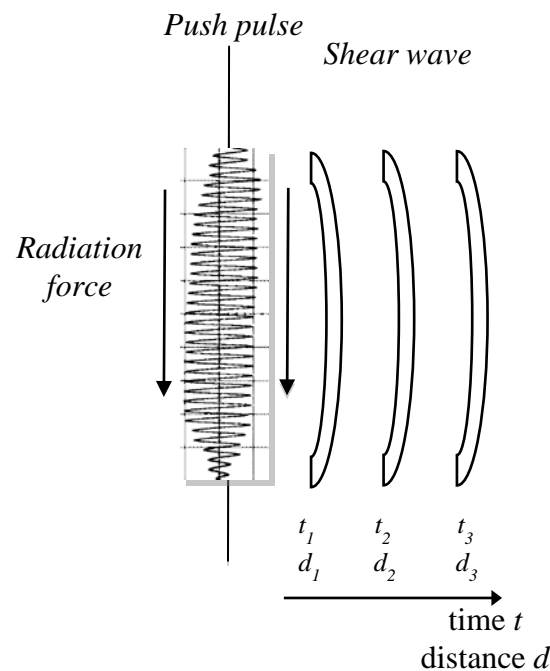


Fig. 2. The radiation force induced by the long push impulse moves the tissue and produces a spherical shear wave.

If the shear wave propagates at 2 m/s, after 10 ms the wave front will move 2 cm. At a resolution of 0.5 mm, the area of transverse dimension 2 cm must be displayed at least 40 times, which translates to 4000 images per second (FR = 4000). In standard ultrasonography FR does not exceed several dozen. Significantly higher imaging rates require the use of new techniques based on synthetic apertures, and especially reconstruction of plane wave images.

In the SuperSonic Aixplorer the push pulses are produced by highly focused ultrasound, and then the shear wave velocity is determined from a 2D image reconstructed with a very large FR. Push pulses are generated by a strongly focused wave in the form of long pulses (several microseconds) with a central frequency of 4 MHz. The sequence of several more pulses is sent by the linear head. Each successive push pulse is focused a little deeper - the speed of the foci is at least a few meters per second greater than the shear wave velocity. In each outbreak a local radiation force is generated, moving the tissue from a few to several micrometers. The local, longitudinal displacement of the tissue produces a shear wave extending perpendicular to the direction in which the subsequent push pulses are produced, Fig. 3. The wave front of the shear waves resembles a Mach cone moving at supersonic speed. So push pulses are moving in the area of a few centimeters (4-6 cm) in about 6 to 8 milliseconds

(repetition rate is about 500 HZ - the speed of the wave source is moving at a speed of 6-7 m/s, hence the name SuperSonic, followed by the sequence of recording e.g. 50 ultrasound images with FR > 5000 and the process is repeated.

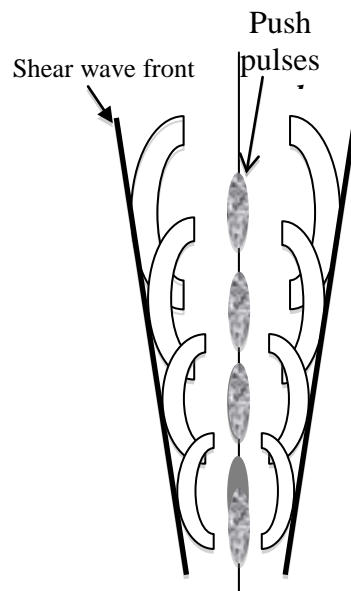


Fig. 3. Shear wave induced by acoustic radiation force in successive push pulse positions. The slope of the shear wave front is proportional to the wave velocity depending on the stiffness of the medium.

The shear wave velocity is determined by the correlation of consecutive images. Once the velocity is determined, the shear modulus is determined, and then Young's modulus is determined. The shear wave velocity in tissues is about a few meters per second, and assuming a purely elastic medium between the shear module and the speed we have a dependency

$$\mu = \rho c_s^2 \quad (15)$$

The assumption is also valid for the visco-elastic model if the dispersion introduced by the viscosity is small and can be neglected.

Figures 4, 5 and 6 show examples of breast and thyroid elastography image registration.



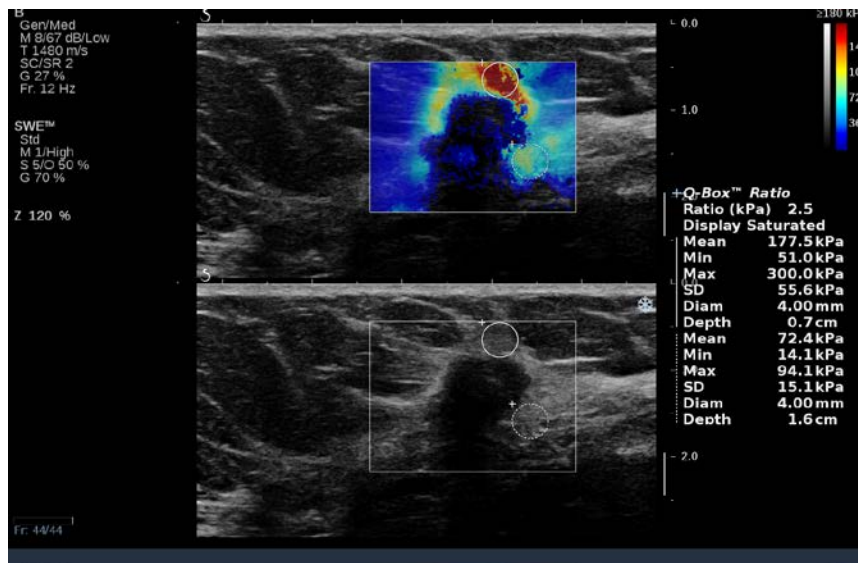


Fig. 4. BIRADS category 4 lesion confirmed by postoperative histopathological verifications as carcinoma tubulare (G2) in dynamic elastography. In the B-mode examination the lesion appears as hypoechoic with indistinct margins, irregular shape and not parallel orientation. Around the lesion a hyperechoic ring is visible. In the elastogram, the surrounding tissue around the lesion is red-coded, E max=300kPa, which is typical for cancer with desmoplastic reaction in the parenchyma.

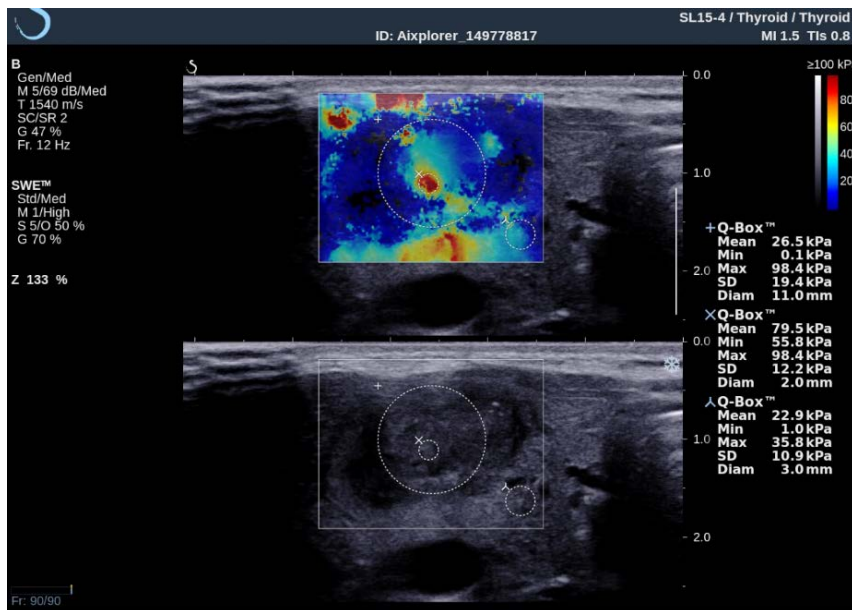


Fig. 5. A solid hypoechoic lesion with microcalcification and indistinct margins, postoperatively classified as medullary carcinoma, is visible in the central part of the left thyroid lobe in the longitudinal section. Elastogram is heterogeneous, with visible hard (in red color) places. SWE elasticity image (the top image) shows three regions of interest (ROIs): „+” refers to the area comprising the largest portion of lesion, „x” refers to a 2-millimeter area from the stiffest part of lesion, „λ” refers to an area of the surrounding tissues with normal thyroid parenchyma. The maximum E value is 98.4 kPa in the 2mm ROI of the stiffest part of the lesion (coded in red – tissues with Young’s modulus  $E > 80$  kPa), and 35.8 kPa in the surrounding tissues.



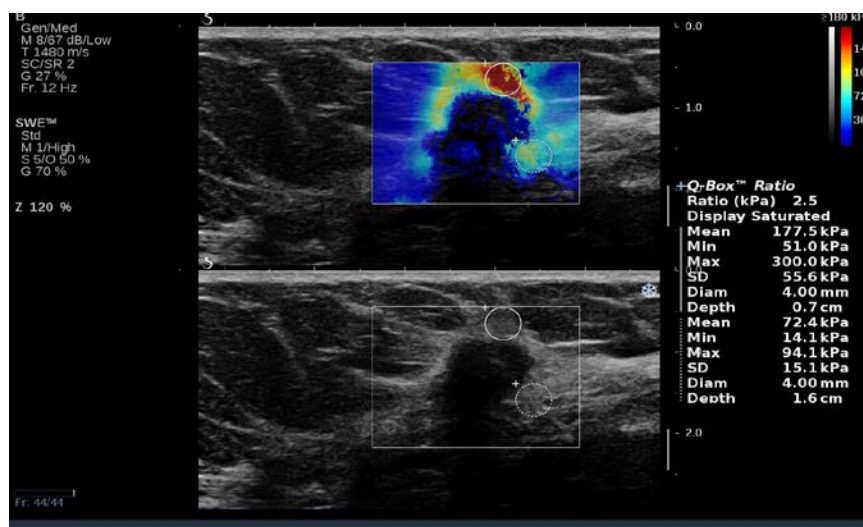


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There are significant differences in the literature between the data in experiments with large deformations. Perhaps they are derived from the computational convention used - the Lagrange model (deformed with respect to the initial length) and Euler (deformed relative to the deformed length). For small deformations differences are not significant.

It is assumed that Young's modulus is about 10 kPa for parenchymal tissue, about 20 kPa for muscles and 50 kPa for the connective tissue. For the rubber, which intuitively appears to be similar to a tissue, the Young's modulus ranges between 900 and 3000 kPa. In the breast, the elasticity modulus is close to 2000 MPa for various types of tissue, while the shear modulus ranges from 20 kPa for adipose tissue to 100 kPa for cancer, [3].

## Discussion

The advent of shear wave elastography, based on tissue displacement under the influence of remotely generated acoustic radiation force is an important part of modern ultrasonography, adding to the quantitative data on local stiffness of tissue lesions.

However, the assumptions of linear, isotropic behavior of tissues and their incompressibility and homogeneity are far from the reality. Various effects of refraction and reflection are neglected in standard ultrasound; however, they substantially alter the image of shear wave propagation. While the differences in longitudinal waves velocities do not exceed 10%, the shear wave velocity can vary in soft tissue up to a factor of ten times, from 1 m / s to 10 m/s. This results in significant reflection and refraction effects due to different acoustic impedances for the shear waves.

In general, the mechanical response of the tissue to the external force depends on the elastic and viscous properties of the tissue, well described by the Kelvin-Voigt model. Elastic materials obey Hook's law, and between the stress  $\sigma$  and the deformation  $\varepsilon$  we have a general relation  $\sigma = E\varepsilon$ . In viscous materials the stress is proportional to the rate of displacement and

is equal to  $\sigma = \mu(d\varepsilon/dt)$ . In the Kelvin-Voigt model the inherent viscosity is parallel to the elastic component. If the deformation is slow, as in the compression (static) method, the time derivative of the displacement is small, and we can neglect the viscosity. In this case, the Kelvin-Voigt model can be approximated only by the elastic component.

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