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Impulse Acoustic Waves: Dynamic Conditions of Unperturbed Propagation

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Secondary radiation from the wave front is calculated in the paper with the use of generalised function approach. The Dirac delta impulse is used as a waveform for both the plane wave and the spherical pressure wave. The results have been obtained analytically thanks to particularly "friendly" features of the operation of convolution with the delta distribution. No approximations have been needed and both solutions are of awaited simple form, i.e.: the original wave displaced forwards - with the proper time delay. Dynamic equilibrium between the elastic and the kinetic aspects of mechanic energy is the only condition of undisturbed propagation of the wave. Every inhomogeneity of the medium reached by the wave breaks the equilibrium, giving birth to a new, scattered wave with features depending on boundary conditions related to the inhomogeneity. The paper gives formal mathematical proof to Fresnel intuitive explanation of the mechanism of forward-only wave propagation with no backward effect.

1. Introduction

Wave phenomena seem to be obvious when regarded from the point of view of mathematics. The plane wave and the spherical one, both fulfil the wave equation. In the case of the plane wave, the solution is assumed to depend on mere one Cartesian coordinate. In the case of the spherical wave, the independence of the two angular coordinates of the spherical system is assumed. Any waveform can be a solution of the wave equation. The only condition is that it should have the second derivative.

When regarded as physical phenomenon, the wave shows well known but somehow mysterious features. First, it participates in transferring the disturbance and energy from one place to another, conserving the previous direction. Second, it does not change the shape in its travel across the medium.

Acoustic waves in liquid media were serving, during over two centuries, as a model for analysing all the wave phenomena. In particular, Huyghens and, later, Fresnel where treating the light as a disturbance in a medium analogous to that of sound in air [4, 3].

Huyghens' hypothesis saying that each point of the medium reached by the wave, becomes the source of a new, secondary wave, seems to be consistent with the experience, as to the field in front of the wave (upstream). However, it fails behind the wavefront, as no new disturbance is observed backwards (downstream) in a homogeneous medium, with no obstacles in it.

For explaining the latter effect, Huyghens postulated particular features of hypothetical secondary sources, namely a directivity depending on the original direction of the primary wave. In fact, his famous graphical construction of the new wavefront defines a one-zero directivity, of Kronecker's delta form, as mere the forward envelope of secondary waves has been taken into account, all the other possible contributions being simply neglected.

The present paper analyses the classical question of mechanisms that are being involved in transfer of the wave disturbance in a medium from one place to its vicinity, according to the Huyghens' idea. It is interesting to look and see how the wave reaching a point "knows" that the propagation should be continued in the previous direction, without changing the shape.

A new question is being raised as well in the paper: In what conditions the secondary wave generated in the medium point is really a ***new*** one?

The analysis starts from the idea of perfect duality of the equations of acoustic. The integral formula equivalent to the classic Kirchhoff formula, with slight modification, is developed in the paper on the basis of straightforward description of the physical nature of action of virtual sources distributed over an arbitrary surface.

Two examples of application of the formula are presented. Secondary radiation from the wave front is calculated with the use of generalised function approach. The Dirac delta impulse used as a waveform for both plane wave and spherical pressure one, represents the most simple and, in the same time, the most general form of a wave. First, such a wave is reduced to its mere front and second, calculations can be generalized to any waveform by the operation of convolution.

2. Fresnel's Hypothesis And Poisson's Doubts

Fresnel gave a very clear description of the propagation mechanism, basing on the duality of the acoustic wave, i.e. on the deformation and the movement of the medium particles. His hypothesis concerned the plane wave. On the one hand, the local overdensity in the wavefront plane results in pushing medium particles in two opposite directions: those in front of the wave being pushed forwards, those behind the wavefront – backwards. On the other hand, the local movement in the wavefront plane results in pushing forwards the medium particles in front and pulling forwards, as well, those behind the wavefront. The superposition of the two effects leads to double the result in front of the actual wave position and to cancel behind. The wave propagates forwards without leaving any wake behind, under condition that both effects are equal.

However, Poisson raised a claim that although the above construction was interesting and, possibly, was not far from the truth, it needed a mathematic proof [2]. First, there it was not sure that both effects were equal and second, even if the Fresnel's explanation were correct for the plane wave, it would not be satisfactory in the case of spherical wave.

3. Duality of Acoustic Phenomena

Acoustic wave is a disturbance in the liquid medium in which coexist and cooperate two basic mechanical phenomena – local change of medium density and local movement of medium particles.

Acoustic pressure is a measure of local density variation, by virtue of the equation of state. Locally unbalanced pressure leads to a local change of the particle movement (Euler's equation). Locally unbalanced stream results in a local density change, i.e. in creation of the pressure (continuity equation).

The acoustic wave equation is a description of mutual actions of deformations and movement in the process of transferring a disturbance across the medium space. The two types of mechanical energy: potential one - related to medium deformation and kinetic one - related to particle movements, have the same weights. In fact, there is no reason to discriminate one of the effects (eg. pressure) at cost of the other. Neither is reasonable to replace both effects by a quantity (velocity potential) pretending to be a universal representation of acoustic phenomena.

Maurice Jessel postulated and realised in his book a non-discriminating approach consisting in equal treatment of acoustic pressure and acoustic velocity [5]. Both quantities have been analysed as dual ones without being replaced by velocity potential, the latter having been treated as mere mathematic quantity without direct physical interpretation.

Dual approach to the acoustic equations leads to two forms of wave equations – one related to the pressure and the other one to the velocity. The solutions of their inhomogeneous forms – in the presence of properly defined respective point sources, give two types of waves of different features: spherical pressure wave and spherical velocity wave. The properties of each wave reflect the physical nature of the source [7].

4. Spherical Pressure Wave

Spherical pressure wave has a scalar quantity as a source, namely the change of the medium matter flow $\rho Q'$ in the source point, where ρ is the medium density in kg/m^3 and Q – the volume flow (source strength) in m^3/s in the source point placed in the origin.

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(\mathbf{x},t)-\nabla^2 p(\mathbf{x},t)=\rho\frac{\partial}{\partial t}Q(t)\delta(\mathbf{x}) \qquad (1)$$

The pressure $p(\mathbf{x},t)$ in the field takes the form of the convolution product of the source quantity and the impulse Green's function $g(r,t)$:

$$g(r,t)=\frac{\delta(t-r/c)}{4\pi r} \qquad (2)$$

that is the solution of inhomogeneous wave equation with, as the second member, an impulse source of no particular physical meaning :

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}g(r,t)-\nabla^2 g(r,t)=\delta(t)\delta(\mathbf{x}) \qquad (3)$$

The waveform of the pressure follows the waveform of Q' in the source point, with the delay due to the distance and with the amplitude decreasing with the distance.

$$p(\mathbf{x},t)=\rho\frac{\partial}{\partial t}Q(t)*g(r,t)=\frac{\rho}{4\pi r}\frac{\partial}{\partial t}Q(t-r/c) \quad (4)$$

The particle velocity accompanying the pressure, calculated from the Euler equation, have the form related to the gradient of the Green's function related to the field point **x**:

$$-grad_x g(r,t)=\frac{\delta'(t-r/c)}{4\pi rc}\hat{\mathbf{r}}+\frac{\delta(t-r/c)}{4\pi r^2}\hat{\mathbf{r}} \quad (5)$$

The velocity being the convolution product of the source strength Q and the gradient (5), it is composed of two terms: the acoustic one – proportional to the pressure, and the hydrodynamic one - decreasing with squared distance, the waveform of which follows that of the source strength :

$$\begin{aligned}\mathbf{v}(\mathbf{x},t)&=\mathbf{v}_a(\mathbf{x},t)+\mathbf{v}_h(\mathbf{x},t)\\&=\frac{\hat{\mathbf{r}}}{4\pi}\left[\frac{1}{rc}\frac{\partial}{\partial t}Q(t-r/c)+\frac{1}{r^2}Q(t-r/c)\right]\end{aligned} \quad (6)$$

The two terms point the radial direction, normal to the wavefront.

5. Spherical Velocity Wave

The source of the velocity spherical wave is related to a vector quantity, namely a variation of the force applied in the source point leading to elastic deformation: $1/Y\,F'$, where Y – medium bulk modulus in kg/s^{2}m and F – the force in kgm/s^2:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{v}(\mathbf{x},t)-\nabla^2\mathbf{v}(\mathbf{x},t)=\frac{1}{Y}\frac{\partial}{\partial t}\mathbf{F}(t)\delta(\mathbf{x}) \quad (7)$$

In this case the particle velocity $\mathbf{v}(\mathbf{x},t)$ has the form of the convolution product of the source quantity and the Green's function $g(r,t)$. The velocity waveform follows, with the delay, that of F', its amplitude decreasing with the distance :

$$\mathbf{v}(\mathbf{x},t)=\frac{1}{Y}\frac{\partial}{\partial t}\mathbf{F}(t)*g(r,t)=\frac{1}{4\pi rY}\frac{\partial}{\partial t}\mathbf{F}(t-r/c) \quad (8)$$

It is worth noting that the velocity vector is no more radial. It points the direction of the source vector.

The accompanying pressure is calculated from the continuity equation as the convolution product of the force F and of the gradient of the Green's function. The result is composed of two terms : an acoustic one – proportional to the velocity and a quasi-static one – following the shape of the force F in the source point. Moreover, the amplitude of both terms depend of the cosine of the angle between the vector F and the radial direction r of the field point :

$$\begin{aligned}p(\mathbf{x},t)&=p_a(\mathbf{x},t)+p_h(\mathbf{x},t)\\&=\frac{\cos(\hat{\mathbf{r}},\hat{\mathbf{F}})}{4\pi}\left[\frac{1}{rc}\frac{\partial}{\partial t}\mathbf{F}(t-r/c)+\frac{1}{r^2}\mathbf{F}(t-r/c)\right]\end{aligned} \quad (9)$$

It is worth noting that the symmetry of the equations and the formulae hide, to some extent, deep difference between both spherical waves, concerning their nature and behaviour [7].

6. Virtual Sources and Superposition of Their Field

The source is a singularity in the field. It introduces an energy to a given space region or absorbs this energy, that leads to its disappearance (in this case it is a drain). In both cases local conditions are modified and a disturbance is created. The stream of the matter and the deforming force introduced by the quasi-point sources are carriers of kinetic and potential energy injected from "exterior" into the infinitesimal region of the medium.

It is usually assumed that the secondary sources postulated by Huyghens, are the virtual surface sources related to the wavefront. The Helmholz and Kirchhoff integral formulae generalize the action of virtual sources onto an arbitrary hypothetic surface $\Sigma(x_o)$ separating two regions of the field [6,1]. In fact, they are not sources *sensu stricto* because they do not inject energy that would be external to the medium. However, when crossing the separating surface, the wave energy disappears in one region and appears in the second one.

Virtual sources distributed on the surface participate in transferring the energy from near to near. It should be admitted that each element $d\Sigma$ of the surface $\Sigma(x_o)$ plays a double role of a source and a drain, depending on the position x of the observer related to the direction of the propagation of the wave.

The present author proposed [8] an approach consisting in attributing to the elements of the hypothetical surface crossed by the wave, source quantities analogous to the quantities attributed to point sources, i.e. volume stream dQ_Σ and force dF_Σ. In fact, the local movement of medium particles and the pressure proportional to local elastic deformation recreate the wave disturbance in the region crossed by the wave.

The volume stream dQ_Σ is defined as a scalar product $vd\Sigma$ of oriented surface element $d\Sigma$ and the velocity v of particles crossing this element. The force dF_Σ is defined as $pd\Sigma$, with p being a local pressure in the vicinity of the surface element.

Secondary sources defined above have a particular feature making them different from primary sources. Their orientation is related to the wave direction of propagation and depends on the observer's position with respect to the hypothetical surface. The effect of the action of each source have to be considered separately in two semi-spaces D^+ and D^- separated by a plane Σ_t tangent to the surface element $d\Sigma$.

The stream of the medium matter should be considered as positive dQ_Σ^+ by the observer placed in the halfspace D^+ in front of the wave, whereas it is negative dQ_Σ^- for the observer placed behind the wave, in the halfspace D^-. Mathematic description should be consistent with the above physical observation, hence the surface element $d\Sigma$ orientation should be defined twofold, by the inversion of the normal vector $\boldsymbol{n}$, as $d\Sigma^+=d\Sigma\,\boldsymbol{n}^+$ and $d\Sigma^-= d\Sigma\,\boldsymbol{n}^-$.

The contributing pressure wave generated by such a source is positive in the downstream region D^+ and negative in the upstream region D^-. It can be written in the form analogous to (4):

$$dp_Q(\mathbf{x},t)=\rho\frac{\partial}{\partial t}dQ_\Sigma^\pm(\mathbf{x}_o,t)*g(|\mathbf{x}-\mathbf{x}_o|,t) \tag{10}$$

In the same way, the vector source will have two "faces". The force dF_Σ due to the local pressure will have the orientation following the orientation of the surface element $d\Sigma$.

The contributing wave generated by this source will be the velocity one. The accompanying pressure can be written as :

$$dp_{\mathrm{F}}(\mathbf{x},t)=-d\mathrm{F}_\Sigma^\pm(t)*grad_x g(|\mathbf{x}-\mathbf{x}_o|,t) \tag{11}$$

In every field point x, the pressure $dp(x)$ is composed of two terms emitted by the doublets of virtual sources distributed over the surface $\Sigma(x_o)$: scalar sources induced by the movement – pure vector quantity, and vector sources induced by the local change of density – pure scalar quantity :

$$p(\mathbf{x},t)=\int_\Sigma\left[dp_Q(\mathbf{x},t)+dp_F(\mathbf{x},t)\right] \tag{12}$$

The total pressure may be expressed in a more explicit form, with r_0 = x-x$_o$ - vector linking the field point with the source element, β - angle between the normal to the surface element $\boldsymbol{n}$ and the velocity vector $\boldsymbol{v}$, γ - angle between $\boldsymbol{n}$ and the direction r_0 of the field point.

$$\begin{aligned}p(\mathbf{x},t)=\int_\Sigma\Big[&\rho\frac{\partial}{\partial t}\mathrm{v}(\mathbf{x}_0,t)\cos\beta(\mathbf{x}_0)*g(r_0,t)\\&+p(\mathbf{x}_0,t)*\frac{\partial}{\partial\hat{r}_0}g(r_0,t)\cdot\cos\gamma(\mathbf{x},\mathbf{x}_0)\Big]d\Sigma(\mathbf{x}_0)\end{aligned} \tag{13}$$

It is easy to show that the above formula can be transformed to the well-known form of Kirchhoff integral formula with, however, one modification. The modification consists in the surface orientation that is neither totally outwards nor outwards but is always towards the observer, i.e. towards the concerned field point.

$$\begin{aligned}p(\mathbf{x},t)=\int_\Sigma\Big[&-\frac{\partial}{\partial\hat{\mathbf{n}}}p(\mathbf{x}_0,t)*g(r_0,t)\\&+p(\mathbf{x}_0,t)*\frac{\partial}{\partial\hat{\mathbf{n}}}g(r_0,t)\Big]d\Sigma(\mathbf{x}_0)\end{aligned} \tag{14}$$

Usually the secondary sources are named monopole and dipole ones that allows us to write :

$$p(x,t)=p_m(x,t)+p_d(x,t) \tag{15}$$

and to calculate separately the monopole and dipole contributions, i.e. movement induced pressure and deformation induced one.

The above approach to the secondary radiation is applied below to the calculation of the classical problem [9] that was the subject of polemics between Fresnel and Poisson [2].

7. Time-domain Impulse Approach

It can be shown that time-domain impulse approach is particularly useful when time-space phenomena are analysed [9]. The convolution product, calculated over a limited time domain, of the Dirac delta distribution and a regular function $A(t)$ gives the result being the original function limited to the domain:

$$\begin{aligned}&\int_{u_m}^{u_M}A_1(u)\delta(t-u)du\\&\int_{-\infty}^{\infty}A_1(u)[H(u-u_m)-H(u-u_M)]\delta(t-u)du\\&=A_1(t)[H(t-u_m)-H(t-u_M)]\end{aligned} \tag{16}$$

Above, H(t) denotes the Heaviside unit step function. When replacing the delta distribution with its time derivative, the result is a linear combination of the function time derivative defined over the respective domain, and two delta distributions related to the function values in moments related to the time domain limits.

$$\begin{aligned}&\int_{u_m}^{u_M}A_2(u)\delta'(t-u)du\\&=\int_{-\infty}^{\infty}(A_2(u)[H(u-u_m)-H(u-u_M)])\delta'(t-u)du\end{aligned}$$

$$=A_2{}'(t)[H(t-u_m)-H(t-u_M)] \qquad (17)$$
$$+[A_2(u_m)\delta(t-u_m)-A_2(u_M)\delta(t-u_M)]$$

The above "friendly" features of the operation of convolution with the delta distribution lead to analytic and exact form of the solutions in the cases where harmonic approach gives mere approximate results.

8. Plane Wave Propagation

The plane wave is a solution of the wave equation when assuming dependence on mere one Cartesian coordinate. Its mathematical form is obvious – the wave of arbitrary shape changes its position with the velocity of propagation. However, the mechanism of propagation from near to near is not clear [5], and another approach is needed when looking for its explanation. Below, such an approach is presented, making use of the secondary radiation from the front of the impulse wave that is "frozen" in a given moment. In fact, such a wave is reduced to its proper front.

The form of the impulse pressure wave crossing the plane $z=0$ is as follows :

$$p(z,t)=P_0\,\delta(t) \qquad (18)$$

The monopole- and dipole-like terms can be expressed, respectively, in the following form :

$$p_m(z,t)=\frac{1}{4\pi}\int_\Sigma \frac{p'(\rho,t)}{c} * \frac{\delta[t-\tau(\rho)]}{r(\rho)}\,d\Sigma(\vec{x}_0)$$
$$=\frac{P_0}{4\pi c^2}\int_0^{2\pi}\int_0^{\infty}\frac{\delta'[t-\tau(\rho)]}{\tau(\rho)}\rho\,d\rho\,d\phi \qquad (19)$$

and

$$p_d(z,t)=\frac{1}{4\pi}\int_\Sigma p(\rho,t) * \left\{\frac{\delta[t-\tau(\rho)]}{r^2(\rho)}\right.$$
$$\left.+\frac{\delta'[t-\tau(\rho)]}{cr(\rho)}\right\}\cos(\vec{n},\vec{x}-\vec{x}_0)\,d\Sigma(\vec{x}_0)$$

$$=\frac{P_0}{4\pi c^2}\int_0^{2\pi}\int_0^{\infty}\frac{\tau_z}{\tau(\rho)}\left\{\frac{\delta[t-\tau(\rho)]}{\tau^2(\rho)}\right. \qquad (20)$$
$$\left.+\frac{\delta'[t-\tau(\rho)]}{\tau(\rho)}\right\}\rho\,d\rho\,d\phi$$

The integration consists in replacing space variables by the time of flight distances and calculating, in the time domain, the integral of convolution with the Dirac delta distribution

The resultant monopole contribution is of a very simple form :

$$p_m(z,t)=\frac{P_0}{2}\int_{\tau_z}^{\infty}\delta'(t-\tau)\,d\tau$$
$$=\frac{P_0}{2}\int_{-\infty}^{\infty}\delta'(t-\tau)H(t-\tau_z)\,d\tau=\frac{P_0}{2}\delta(t-\tau_z) \qquad (21)$$

The dipole contribution being a bit more complicated, the final result is the same.

$$p_d(z,t)=\frac{P_0}{2}\int_{\tau_z}^{\infty}\left[\frac{\tau_z}{\tau^2}\delta(t-\tau)+\frac{\tau_z}{\tau}\delta'(t-\tau)\right]d\tau$$
$$=\frac{P_0}{2}\left[\frac{\tau_z}{t^2}H(t-\tau_z)+\left(\frac{\tau_z}{t}H(t-\tau_z)\right)'\right] \qquad (22)$$
$$=\frac{P_0}{2}\left[\frac{\tau_z}{t^2}H(t-\tau_z)-\frac{\tau_z}{t^2}H(t-\tau_z)+\frac{\tau_z}{t}\delta(t-\tau_z)\right]$$
$$=\frac{P_0}{2}\delta(t-\tau_z)$$

It should be noticed that the condition *sine qua non* for obtaining such a simple result, is to maintain every term during the whole analysis and not to apply usual approximations (eg. neglecting terms decreasing with squared distance).

It is not difficult to show that the two contributions having the following signs in the upstream region:

$$p_m(z^-,t)=-p_m(z^+,t)$$
$$p_d(z^-,t)=+p_d(z^+,t) \qquad (23)$$

compensate each other behind the wavefront:

$$p(z^-,t)=p_m(z^-,t)+p_d(z^-,t)$$
$$=\frac{P_0}{2}\delta(t-\tau_z)-\frac{P_0}{2}\delta(t-\tau_z)=0 \qquad (24)$$

Fig.1 illustrates the phenomenon of one way, forward-only propagation, without any wake behind.

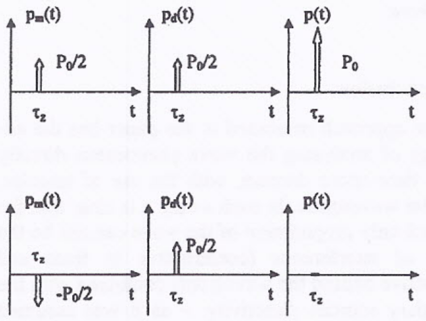

Fig. 1. Monopole- and dipole-like contributions, and the total pressure in the field points **x**: *a) in front of the impulse wavefront and b) - behind it.*

9. Spherical Wave Propagation

The same method can be used to calculate the field in front of and behind the front of spherical wave, "frozen" in a given moment.

$$p(\vec{x},t)=\frac{P_s}{|\vec{x}|}\delta\left(t-|\vec{x}|/c\right) \quad (25)$$

The calculations of the field are longer in this case, and the details can not be presented here. However, the results remain relatively simple, viz.:

$$\begin{aligned}p_m(t)=&\frac{P_s}{2z}\left\{\pm\frac{1}{\tau_0}\left[H(t-u_m)-H(t-u_M)\right]\right\}\\&\pm\frac{P_s}{2z}\{[\delta(t-u_m)-\delta(t-u_M)]\}\end{aligned} \quad (26)$$

$$\begin{aligned}p_d(t)=&\frac{P_s}{2z}\left\{\mp\frac{1}{\tau_0}\left[H(t-u_m)-H(t-u_M)\right]\right\}\\&+\frac{P_s}{2z}\{[\delta(t-u_m)\mp\delta(t-u_M)]\}\end{aligned} \quad (27)$$

The above results are illustrated in Fig..2.

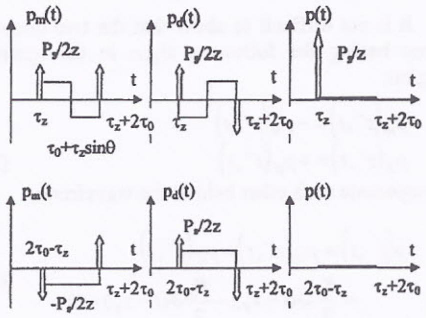

Fig. 2. Monopole and dipole-like contributions, and the total pressure in the field points x a) upstream and b) downstream the spherical impulse wavefront.

10. Conclusion

The approach presented in the paper has the advantage of analysing the wave phenomena directly in the time-space domain, with the use of impulse, unipolar waveforms. In such a case it is clear that the forward-only propagation of the wave can not be the result of interference (constructive in front and destructive behind the wavefront) combined with the secondary sources directivity. – as it was assumed for harmonic waves by Huyghens, admitted, finally, by Fresnel, and is still suggested in textbooks.

Fresnel's hypothesis remaining crucial for understanding the mechanism of the forward-only wave propagation, it has been proved by the present author.

It has been demonstrated that the conservative behaviour of acoustic wave (conservation of the direction of propagation and of the shape) is due to the dynamic balance between elastic deformation of the medium and inertia of particle movement.

The waves as the product of secondary radiation from their proper front – these original results conform the intuitive description given by Fresnel and are the mathematical proof asked for by Poisson, covering both plane and spherical wave case. Probably, construction of such a proof was impossible before the development of the calculus of distribution [10]. In any case, no such proof has been found in the frame of harmonic approach.

Moreover, it becomes clear that *new* wave postulated by Huyghens appears in these points only, where the dynamic balance between two types of secondary sources is broken and one type of sources dominates over the other. Otherwise speaking, really new waves - scattered or reflected ones, are created *always and only* at boundaries of medium inhomogeneity.

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