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Inverse Problems in Underwater Acoustics

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The paper presents an overview of the inverse problems associated with applications of underwater acoustics. The problems are classified into four main categories, according to the recoverable quantities: Acoustic source localization, bottom identification, ocean acoustic tomography and target recognition. The paper gives emphasis to the three first categories, and includes methods to solve the inverse problems associated with the appropriate experimental configurations.

1. Introduction

Inverse problems in underwater acoustics have recently drawn the attention of scientists working in underwater technology, due to the relatively high efficiency of the sound waves as carriers of information related to the environmental and operational parameters in the sea environment, including acoustic source characteristics and shapes of objects in the water or the bottom layers.

The recovery of these parameters using measurements of the acoustic field is the main objective of the inverse problems in underwater acoustics

According to the classification proposed by Collins and Kuperman [1], the inverse problems in underwater acoustics fall in two main categories: Remote sensing of the sea environment and localization. In the first category problems such as that of estimating sound speed structures and current fields in the water column (acoustic tomography), bottom properties and roughness parameters are encountered while in the second one, problems of source recognition and localization as well as source path estimation are faced. In this paper we will adopt a more fine classification, by dividing each one of these two categories into two more. Thus, remote sensing is divided in ocean acoustic tomography and bottom identification, while localization is divided into source localization and target recognition. This division is mainly dictated by the actual methods developed for solving the corresponding problems.

The structure of the paper is as following: In the first section, general remarks on the inverse problems in underwater acoustics will be made. In the subsequent sections, problems from the three first categories will be presented, together with a selection of methods developed for their treatment and some typical illustrative examples.

2. Formulation of the inverse problems

Conceptually, the formulation of the inverse problems is simple. Practically, all the inverse problems we encounter in underwater acoustics are discrete. A set of model parameters

$$\mathbf{m} = [m_1, m_2, ..., m_M]^T$$

is to be recovered from a set of measured data **d**.

$$\mathbf{d} = [d_1, d_2, ..., d_N]^T$$

Model parameters and data are related through a generally non-linear vector equation of the form:

$$\mathbf{f}(\mathbf{m}, \mathbf{d}) = 0 \tag{1}$$

The equation is determined by appropriate modeling of the forward acoustic propagation problem and in general function **f** is complicated and not amenable to linearization. Thus, the problem is non-linear and very difficult to be solved. In some cases though, the problem can be linearized, and is reduced to the solution of a linear system of equations of the form:

$$\mathbf{d} = \mathbf{G}\mathbf{m} \tag{2}$$

Written analytically as

$$d_i = \sum_{j=1}^{M} Q_{ij} m_j, \qquad i = 1, ..., N \tag{3}$$

The system can be underdetermined, even determined or overdetermined according to the number of the available data in association with the number of the recoverable parameters. For an overdetermined system, which is frequently the case, least square solutions are usually derived. There is no need to go into details on the solvability of the system or in general equation (1). It is well known that inverse problems based on discrete measurements of field data are ill posed and the uniqueness or even the existence of the solution cannot be easily ensured.

In summary, the key factor governing the solvability of an inverse problem is the determination of the function **f**. As the data and the parameters can be associated by different ways, it is up to the researcher to decide the appropriate way, which in turn will be dictated by the existence of the necessary means to perform the measurements and of course the inverse methods he has in mind.

In the subsequent sections we will refer to some standard inverse problems in underwater acoustics, presenting a selection of methods that have been proposed so far for their treatment. The presentation of the methods is far from being complete, but it will give an overview of the subjects of current research in the field. It has to be noted that the methods to be presented are used by the group of the Institute of Applied and Computational Mathematics in FORTH, working with inverse problems in underwater acoustics.

3. Ocean acoustic tomography

Ocean acoustic tomography has been introduced in underwater acoustics as an alternative method for the remote sensing of the oceanographic processes, through acoustic measurement [2]. Since the first time that this concept drew the attention of the acousticians and the oceanographers, a number of methods for solving the associated inverse problem have been developed and the feasibility of the concept itself has been demonstrated by field experiments.

Ocean acoustic tomography is traditionally referred to the recovery of the temperature structure of the sea, through measurements of the acoustic field. The recovery is made indirectly in the sense that it is the sound speed, which is directly recovered, the temperature being estimated through semi-empirical formulae.

3.1 Ray inversions

Originally, ocean acoustic tomography was based on ray theory, since the vehicle, carrying the information on the environment was the acoustic ray. It was the ray travel time, which gave the necessary information for the calculation of the sound speed. Broadband sources were used and the signal was recorded at a single hydrophone.

Linearizing the problem with respect to a known "background state", travel time variation $\delta\tau_i$ along a certain ray Γ_i is associated with sound speed variation through the formula

$$\delta\tau_i = \int_{\Gamma_i} \frac{\delta c(\vec{x})}{c_0^{\,2}(\vec{x})} ds \tag{4}$$

where, c_0 is the sound speed for the background state and $\vec{x}$ are the spatial variables.

Provided that ray arrivals could be identified in the receiver location, the use of N measurements could result in the extraction of the sound speed along the ray path and finally at discrete points in the water column. The problem is normally solved by discretization of the ray path and the use of orthogonal functions to describe the sound speed variations (see below).

The method has been extensively used especially in deep-water areas with good results. It should be noted that tracking of the peaks, that is relating the peaks of the signal of the background environment with them of the actual measurements, is essential for the application of the method.

3.2 Modal travel time inversions

An alternative approach is to identify modal arrivals instead of rays. This approach could be applied in shallow water areas where there is better modal resolvability with respect to the deep water case. Modal travel time is defined as the travel time of a modal packet propagating in the water column. The modal velocity is defined as:

$$v_{gn} = \left. \frac{\partial \omega}{\partial k_n} \right|_{\omega_0} \tag{5}$$

where k_n is the eigenvalue of order n, when the problem is solved using normal-mode theory.

The formula associating modal travel time variations with respect to a background environment is of the form:

$$\delta\tau_n = \int_S \left. \frac{\partial Q_n}{\partial \omega} \right|_{\omega_0} \delta c(\vec{x}) d\vec{x} \tag{6}$$

and the integration is over the area of the sound speed variation. The function Q_n is calculated for the parameters of the background environment [3,4].

By appropriate discretization of the water environment, the application of this formula for N measurements of the modal travel time, result in the linear system

$$\boldsymbol{\delta\tau} = \mathbf{G}\boldsymbol{\delta}\mathbf{c} \tag{7}$$

Note that the application of formula (4) under a similar discretization process leads to same system of equations.

The method works well for the recovery of range-average sound speed profiles or structures where the recoverable parameters are local variations described by means of empirical orthogonal functions (EOFs). EOF's are functions determined by statistical analysis of historical data and are frequently used to describe the depth variation of the sound speed.

$$\delta c = \sum_{\ell} \theta_{\ell} f_{\ell}(z) \tag{8}$$

Here, θ_ℓ is the amplitude of the EOF.

It has been shown that the use of EOFs has a twofold effect as regards the inverse problem. First it leads in a decrease of the number of the unknowns (normally 2 or 3 orders of EOFs are enough to determine the field), thus rendering the problem more easy and second ensures the physical meaning of the results by forcing them to be smooth functions.

Figure 1 presents an example of EOFs (three orders) determined for the environment of the Gulf of Lions in France. They are scaled for simulation reasons to the depth of 400 m. They correspond to the sound speed variation with respect to a linear reference profile.

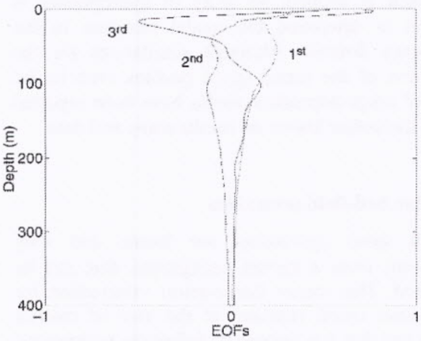

Fig. 1. Empirical Orthogonal Functions

Using EOFs, equation (7) is referred to the amplitudes θ_ℓ of the EOF's *instead* of the sound speed values c_j

$$\boldsymbol{\delta\tau} = \mathbf{G}_1\boldsymbol{\delta\theta} \tag{9}$$

When inverting for a range-dependent environment the amplitudes are functions of range.

3.3 Peak inversions

When neither ray arrivals nor modal arrivals are identifiable, an alternative approach has been introduced allowing inverting for the local maxima of the arrival pattern of the tomographic signal. If the amplitude of the pressure at the receiver's location is

$$a(t; c(\vec{x})) = a(t; \vec{\theta}) \tag{10}$$

the arrival times of the local maxima satisfy the equation

$$a'(\tau_t; \vec{\theta}_0) = 0 \tag{11}$$

where prime denotes differentiation.

Using this approach, a linear relationship between travel time variations of the peaks and associated sound speed variations in the same way as in equation (7) can be defined. [5]

This approach has been used extensively for processing the data of the THETIS I and II experiment in the Mediterranean Sea with excellent results [6,7]. Range average sound speed profiles have been obtained so far, but progress is underway to assess the conditions of applicability in range-dependent environments.

3.4 Modal-phase inversions

A similar approach is to invert for the amplitudes of the EOFs or the sound speed values using measurements of the modal-phase differences, defined as the phase of each normal mode filtered at the receiver's location. However, in order that this approach is applied, an array of hydrophones is needed to determine the modal structure in the frequency domain. Although simulations for the inversion of the sound speed profiles even in the case of range-dependent media have been reported [4,8], the author knows no results using real data.

3.5 Matched-field inversions

All these approaches are linear, and only variations from a known background date can be obtained. This means that a-priori information for the sound speed structure at the area of interest exists and that it is enough to define the background environment. When this is not the case, non-linear schemes have to be applied. So far, inversions based on matched-field processing are very popular, despite the limited number of experiments that have been performed for their validation.

The sound speed structure at a specific area is determined by "matching" replica fields with measured ones, the replica fields determined by means of a suitable direct propagation model. The matching over a generally wide search space is controlled by means of an objective function, which has its maximum when its input set consists of the real model parameters [9].

Thus, the solution of the problem is in principle obtained by looking for the model parameter vector $\mathbf{m}$ that maximizes the objective function $L(\mathbf{m})$.

The approach is very simple in its concept but it is computationally expensive, due to the great number of estimations for the replica fields that have to be performed. Of course, the process is controlled by a suitable algorithm aiming at reducing the number of the required calculations. This can be done by directing the search towards most probable solutions or to a population of acceptable solutions. Simulated Annealing [9], Genetic Algorithms [10] are typical examples of algorithms developed to control the directive search. Hybrid approaches combining non-linear and linear techniques have also been applied [12].

Figures 2 to 4 present examples of the use of such a hybrid approach using simulated data. The approach is based on the use of a background environment for a modal-phase approach, which has been the result of the application of a matched-field scheme. Figure 2 is the actual environment corresponding to a cold eddy, described by means of EOFs of the type of Figure 1. Figure 3 presents the recovered structure, when a matched-field processing scheme with a Genetic Algorithm is applied and Figure 4 corresponds to same environment recovered by a hybrid scheme.

The objective function used in this matched-field processing was a standard Bartlett processor defined as

$$P(\mathbf{m}) = \mathbf{w}^{+} C \mathbf{w} \tag{12}$$

where $\mathbf{w(m)}$ is referred to the calculated pressure field and $C = \boldsymbol{\alpha}\boldsymbol{\alpha}^{+}$, and $\boldsymbol{\alpha}$ is referred to the meassured pressure field

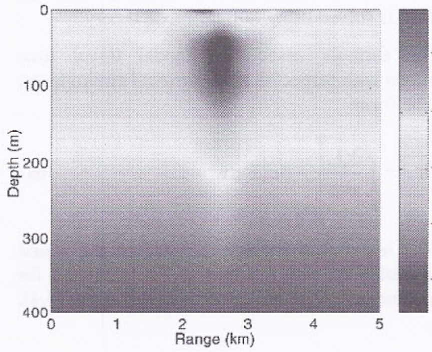

Fig. 2. The simulated structure of a cold eddy

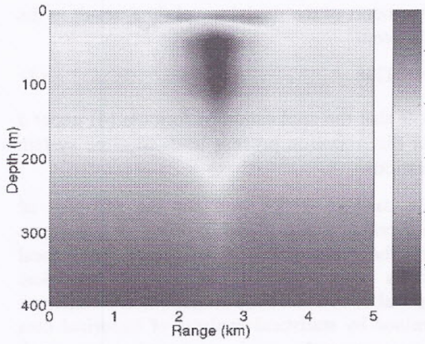

Figure 3. The structure of the cold eddy recovered using matched-field processing

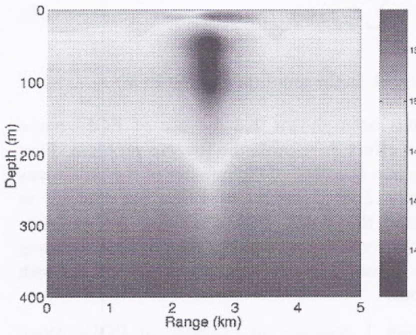

Fig. 4. The structure of the cold eddy recovered by the hybrid approach

Alternative approaches in the context of non-linear schemes include matched-mode schemes in which the sound speed structure is calculated by matching the modal structure rather that the pressure field [13], and neural networks [14] in which the field matching is preceded by a learning phase, aiming at teaching the network to understand the differences in the fields and a possible reason for these.

4. Bottom classification

The classification of the sea-bottom by acoustic means has been a subject of research since a long time. The reasons are obvious. Knowledge of the nature of the sea bottom is an important factor governing all applications of underwater-acoustics especially in shallow water areas. Two main classes of inverse methods could be mentioned at this point: Ray theoretic approaches and full wave techniques. In the first class belong all methods that decompose the acoustic field into plane-wave components and use the concept of reflection coefficient as a vehicle for performing the necessary inversions. All the methods using measurements of the acoustic field without making reference to its ray composition fall in the second class.

It should be noted that ambient noise is an alternative source of information for imaging the sea environment which of course includes the estimation of the bottom parameters [15]. It is well known that the spatial characteristics of the ambient noise are correlated with the ocean sound speed structure in both the water column and the bottom. Accordingly, if there is a good knowledge of the water parameters at a specific region, which is often the case, bottom parameters can be estimated by exploiting the ambient noise information obtained using arrays of hydrophones. We will not go into details for this approach. Instead we will present here two approaches corresponding to the first two classes of methods which are supported by field and simulated data.

4.1 Ray-theoretic inversions

Ray theoretic methods are more appropriate when the acoustic measurements are made at a location very close to the source (local methods). In these cases it is sometimes more convenient to assume that the acoustic field can be accurately described as a superposition of plane waves and apply inversion schemes based on the extraction of the plane wave reflection coefficient from the actual measurements. Normally, broadband sources are used and the inversions are performed in the frequency domain.

A typical experimental set-up is shown in Figure 5. It corresponds to the setting of the REBECCA and SIGMA projects, which are funded by the EU [16]. The source is a parametric array and the measurements are taken at the hydrophones of a streamer towed behind the sonar at the same depth.

If measurements of the reflected field for a number of different angles of incidence are made, the inversion procedure is structured in such a way that exact knowledge of the source excitation function is not needed. However, exact geometry of the source receiver system including bottom topography has to be known.

The idea is to calculate reflection coefficient ratios between different angles of incidence

$$I(\theta_i,\theta_j) = \frac{R(\theta i;\omega)}{R(\theta j;\omega)} \tag{13}$$

It can be shown that under the geometry of the experimental set-up, this ratio is close to the ratio of the reflected fields

$$I'(\theta_i,\theta_j) = \frac{S(\theta i;\omega)}{S(\theta j;\omega)} \tag{14}$$

Thus, it is possible to use any frequency within the source bandwidth to determine ratios I' which of course is a function of the bottom parameters, that is, for the case of a single layered bottom, the shear and compressional velocities and corresponding attenuation as well as the bottom density. For a multilayered bottom the unknown coefficients include the parameters of all the layers along with the thickness of the layers.

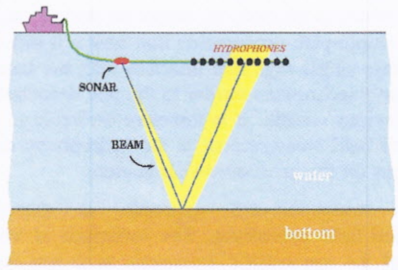

Fig. 5. Experimental set-up for bottom recognition using a parametric array source

Using M angles of incidence we obtain M-1 values of corresponding ratios forming a system of non-linear equations that can be solved for the unknown geoacoustic parameters **m**.

The system thus determined can be written in vector notation as:

$$\mathbf{f}(\mathbf{m},\mathbf{d}) = 0 \tag{15}$$

Since the system is non-linear, various techniques could be used for its solution, including local search algorithms based on the assumption that there is a good indication of the actual field.

An example of the use of this method is given below. Input data are from an experiment held in August 1994 in the Atlantic Ocean close to the French coast near Brest at a site with average water depth of 70 m. A horizontal array of hydrophones was towed behind a parametric array mounted at a submersible vessel (fish) . The aperture of the array was 48 m and the spacing between the 25 hydrophones was 2 m. The whole system was towed by the French research vessel "*Le Noroit*".

The signals received at the streamer were filtered to eliminate noise and the precise geometry of the experiment was determined using navigational data for the whole system.

Results of inversion for two different sites within the area of the experiment (49 and 51) are presented in Table I. The angles of incidence used were in the range between 30 and 35 degrees which cannot be considered optimum for the type of bottom expected in the area, given the information on the bottom material (silt), being away from the critical angle. This is why no attenuation coefficient could be obtained. However the results are clearly satisfactory.

Table I: Inversion results

Parameter	Initial Value	Result (49)	Result (51)	Average Values (silt)
P_2 kg/m^3	1500	1269	1245	1280
c_{p2R} (m/s)	1800	1502	1518	1490
$C_{s2R}(m/s)$	500	70	84	180
A_p (dB/λ)	0.0	-	-	0.09
A_s (dB/λ)	0.0	-	-	0.09

4.2 Wave theoretic inversions - matched field processing

If the array is positioned away from the source, local reflection phenomena cannot be studied, due to the multiple-path propagation that makes it difficult to isolate single rays that interfere with the bottom and apply techniques similar to the one described in the previous section. It is therefore desirable to use the full-field measurement at the hydrophone array to invert for the geoacoustic parameters.

A straightforward approach is therefore matched-field processing. The technique is vastly applied for geoacoustic inversions as it was noted during a recent benchmark exercise for bottom geoacoustic inversions (Vancouver 1997), where most of the presentations were based on this technique, the only differences being in the search algorithm utilized. In this context genetic algorithms were proven to attract the interest of the majority of the participants [17].

In this section a matched field algorithm is presented along with the results obtained for one of the benchmark cases of the experiment mentioned above [18].

An interesting feature of the benchmark exercise was that broadband simulated data were available to the participants. If a matched-field processing algorithm is to be applied, the objective function has to be defined. Four alternative cost functions are mentioned below: They have been proven to give similarly good results for the problem in hand the difference among them being mainly in the way that the processing is performed in the frequency domain [18]:

COH1

$$L_1(m) = \frac{1}{I}\sum_{i=1}^{I}\left(\left|\sum_{p=1}^{N}\sum_{p=q+1}^{N}\sum_{B} D_{pq}(f) M^*_{pq(m)}(f)\right| K^{-1}(m)\right)_i$$

COH2

$$L_2(m) = \frac{1}{I}\sum_{i=1}^{I}\left(\left|\sum_{p=1}^{N}\sum_{p=q}^{N}\sum_{B} D_{pq}(f) M^*_{pq(m)}(f)\right| K^{-1}(m)\right)_i$$

INCOH1

$$L_3(m) = \frac{1}{I}\sum_{i=1}^{I}\left(\sum_{B}\left|\sum_{p=1}^{N}\sum_{p=q+1}^{N} D_{pq}(f) M^*_{pq(m)}(f)\right| K^{-1}(m)\right)_i$$

INCOH2

$$L_4(m) = \frac{1}{I}\sum_{i=1}^{I}\left(\sum_{B}\left|\sum_{p=1}^{N}\sum_{p=q}^{N} D_{pq}(f) M^*_{pq(m)}(f)\right| K^{-1}(m)\right)_i$$

where B denotes the frequency band,

$$D_{pq}(f) = D_p(f) D_q^*(f) \tag{16}$$

is the data cross spectrum, and

$$M_{pq(m)}(f) = M_{p(m)}(f) M^*_{q(m)}(f) \tag{17}$$

is the modeled cross spectra and the star (*) denotes complex conjugate.

D_p is the complex pressure measured in the hydrophone p and $M_{p(m)}$ is the complex pressure calculated by a suitable algorithm solving the forward propagation problem for the candidate model vector $\tilde{\mathbf{m}}$.

$K(m)$ is a normalization constant defined so that the objective function takes its maximum value 1 when the candidate model vector $\tilde{\mathbf{m}}$ is identical to the actual model vector **m**.

The environment considered here is one of the test environments of the benchmark exercise and it is denoted as WA. Its basic properties appear in figure 6 and consist of a water layer over a two-layer bottom of fluid material. It is interesting to note that according to the specifications of the experiment, the water depth as well as the source

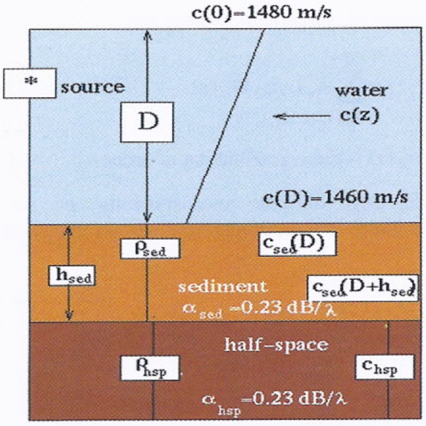

Fig. 6. The test environment for bottom geoacoustic inversions

location were additional unknowns to the bottom parameters. The sound velocity structure in the water was known as well as the attenuation in the bottom layers. The problem had therefore 9 unknowns to be recovered.

Data provided by the organizers of the benchmark exercise that included noise, and calculated in a number of frequencies (from 25 to 199 Hz were used. An experiment with a vertical linear array placed at a nominal distance of 4 km, from the source, spanning 100m in water depth with hydrophone spacing 1 m was simulated. All the objective functions mentioned above were tasted and an a-posteriori statistical analysis has been used to predict statistical means and moments for the recoverable parameters, following Gerstoft [19]. Replica fields were calculated using the standard propagation code SAFARI [20]. A Genetic Algorithm described in [12] was used and a total of 10.000 models were computed. The best 100 of them were kept for the statistical analysis.

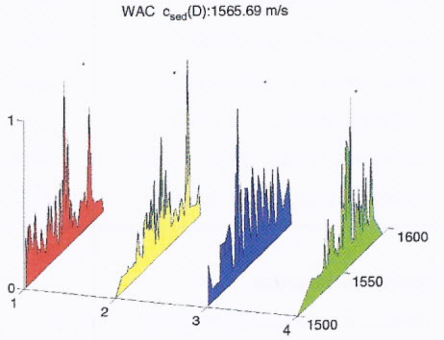

Fig 7. The a-posteriori marginal probability function for $c_{sed}(D)$.

Figure 7 presents an example of results obtained by the statistical analysis. They correspond to the sound speed at the upper part of the sediment layer. The actual values in the figure are marked with a star. It can be easily seen that the value corresponding to the maximum probability for each one of the parameters is very close to the actual value. Also, we note the rather equivalent behavior of the various objective functions with the only significant difference in the results coming from the incoherent processing in the frequency domain that can be considered as more reliable. This can be partly explained by the fact that since we had data from tones (and not a real broadband source), the shape of the signal could not be exploited, in which case the phase in each frequency would definitely provide more information for the inversion procedure.

Finally, Table II presents the values of the geoacoustic parameters, as well as the water depth and the source position as calculated by the matched-field processing scheme and corresponding to the model that gave the maximum fit, that is the model vector resulting in the maximum value of the objective function.

It can be easily confirmed that values corresponding to the parameter set showing best fit with respect to given data, are very close to the actual values of the parameter set. It should also be noted that various modifications including two-stage processing which exploits the different effect that lower frequencies have with respect to higher frequencies lead to similar results [21].

Table II. Inversion results using matched-field processing. Values corresponding to best fit

Parameter	COH1	COH2	INCOH1	INCOH2	True value
Source range (m)	-279.4	-273.0	-273.0	-298.4	-290.0
Source depth (m)	28.06	28.06	28.06	28.06	28.23
Water depth (m)	119.68	119.36	120.00	120.00	119.88
Sediment thickness, h(m)	30.95	27.78	30.47	29.68	28.95
Sound speed, $c_{sed}(D)$ (m/s)	1579.	1579.	1562.	1565.	1565.69
Sound speed, $c_{sed}(D+h)$ (m/s)	1587.	1587.	1588.	1616.	1591.76
Sediment density, ρ (g/cm^3)	1.63	1.72	1.67	1.71	1.68
Subbotom sound .speed c_{hsp} (m/s)	1678.6	1698.4	1703.1	1706.3	1707.12
Subbottom density, ρ (g/cm^3)	1.78	1.80	1.89	1.89	1.88

5. Source localization

Source localization is referred to the problem of identifying the position of a sound source. Most of the methods that have been proposed so far exploit matched-field or matched-mode processing algorithms. As a matter of fact matched-field algorithms have been introduced as a tool for source localization. Necessity for source localization problems emerges in practically all the applications of underwater acoustics. This is due to the fact that the exact position of the source is an indispensable information for solving large classes of inverse problems, including ocean acoustic tomography and bottom recognition. The matching techniques have the advantage that been non-linear by their nature could be used for the recovery of a multparameter space, which of course could in principle include the information on the location of the sound source. Thus, source localization was a sub-product of the bottom recognition mentioned above, as indicated in the Table II. It is interesting to present the corresponding statistical distributions for the experiment mentioned above (Figure 8).

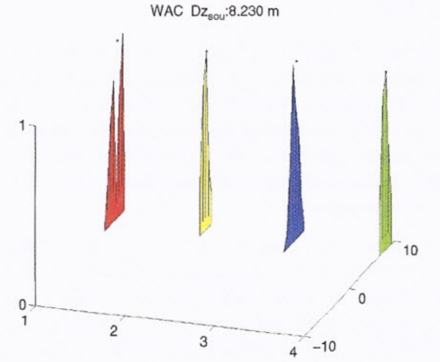

Fig. 8: The a-posteriori marginal probability function for the source depth.

The next example gives results for source localization in a different way. They have been obtained using matched-mode approach. A vertical array of hydrophones is used to measure the acoustic field.

In brief, matched-mode approach as applied at FORTH, is based on the recovery of the eigenforms $A_n(r;z_0)$ of the modal expansion of the acoustic field written in cylindrical co-ordinates:

$$p(r,z)=\sum_{n=1}^{N} A_n(r,z_0)u_n(z) \qquad (18)$$

where $u_n(z)$ is the eigenfunction of order *n*.

Performing discrete measurements of the acoustic field in J hydrophones, we get the data vector:

$$\mathbf{d}=[p_1,p_2,.....p_J]^T \qquad (19)$$

where,

$$p_j=\sum_{n=1}^{N} u_{jn}A_n, \qquad j=1,2,...,J$$

The "model" vector **m**, is defined as

$$\mathbf{m}=[A_1,A_2,.....A_N] \qquad (20)$$

and can be obtained by solving the system defined in by least square method

$$\mathbf{m}=(U^TU)^{-1}U^T\mathbf{d} \qquad (21)$$

provided that the matrix $(U^TU)^{-1}$ is invertible.

To proceed, we need a means to "match" the eigenforms A_n. This can be achieved by defining an appropriate cost function in the same way as in the case of matched-field processing. Again, we have to apply suitable search algorithms for accelerating the convergence of the procedure.

In the example presented in Figure 9, we have used the environment WA of the benchmark exercise mentioned in the previous section. An array with just 25 hydrophones spaced 4 m apart have been used covering the first 100 m in the water depth at the nominal distance of 4 km. A simple search algorithm was applied with a cost function being an adaptation of the Bartlett processor used for matched-field processing. In doing so, $\mathbf{w}$ and $\boldsymbol{\alpha}$ are referred to the complex eigenform instead of the complex pressure.

The search space (20-35 m in depth and 3000-4500 in range) was discretized in small range and depth increments and all possible values were checked through the program MODE1 developed by the author. The results showing good source localization appear in figure 9. The figure presents the cost function calculated using the corresponding range and depth for the sound source. Remember that the actual source range is 3790 and the actual source depth is 28.23 m.

6. Discussion

Various classes of inverse problems appearing in applications of underwater acoustics have been presented along with characteristic methods for their solution. The formulation of the problems was based on discrete inverse theory, while the methods for their treatment were either linear or non-linear. Most of the methods were supported by inversion simulations, which were chosen in such a way so that they can represent actual situations and in one case with inversions corresponding to real data. Both ray and wave-theoretic approaches were considered, but no attempt was made to evaluate the various approaches. This is a rather difficult task since the potential of a specific method has to be considered in connection with the actual experimental conditions and the quality of existing data. In general, matched-field or matched-mode methods behave in a better way as compared with linear schemes when the available data are noisy.

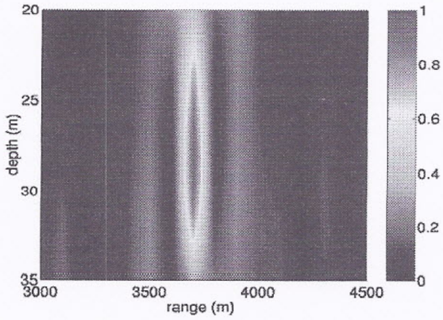

Fig 9: Source localization for the environment of presented in section 4.2 using matched-mode processing.

On-going research in the field is currently focused on inversions using broadband data and fast algorithms and on inversions performed in a range-dependent environment. Range-dependent inversions are essential when dealing with long range monitoring of the ocean environment, while preliminary results on the use of the time information as complimentary to space information has shown to improve the accuracy of the parameter estimations.

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