

Spectrum Analyzers for Nonlinear Phenomena Investigations.

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The subject of the paper is the application of digital signal processing (DSP) to nonlinear hydroacoustic, acoustic and mechanic phenomena investigations. To provide such investigations a spectrum analyzer with additional functions of polyspectra and virtual input channel is proposed. The mathematics, equipment and programming of the analyzer are discussed. It is shown how the analyzer enables to detect and to explore the random impulse sequences combined with one another and with Gaussian noise, to separate the oscillations having close unstable frequencies, to observe the nonlinear interaction between oscillation modes, to detect the noise emission accompanying the oscillation mode energy dissipation.

1. Introduction.

The digital signal processing (DSP) is widely applied to study mechanical systems oscillations, sound and vibrations waves. The spectrum analysis is one of the efficient methods of the DSP; sometimes it aims at the gaining the information about state of mechanical system or environment where oscillations are generated or propagated. The spectrum analysis method is especially efficient when the stationary linear systems (due to their capability to pass harmonic oscillations undistorted both in form and frequency), and the Gaussian signals (because of the statistical independence of their frequency components) are investigated. There are many devices such as spectrum analyzers which support the measurement of auto- and cross-spectra, and on the basis of these accomplish the computation of correlation, coherence and transfer functions. The above mentioned characteristics of signals and systems would be called "classic" as distinct from some others, more seldom used at present, which would be discussed below. The classic characteristics are of remarkable use for researchers for a number of reasons: what is most important they are adequate in description of Gaussian signals and linear systems; being the function of one variable (frequency or delay) they are easily displayed and interpreted, and are adequate for the estimation of mechanical and biological effect. The high speed spectrum analyzers used for the measurements are capable of operation with the

"live signal", i.e. they provide the study of quasi-stationary signals, the monitoring of their parameters and other similar applications. Moreover the "classic" analyzers are comparatively cheap and compact. Still the set of classic spectrum analyzers does not allow to investigate the oscillations in nonlinear mechanical systems property. In particular, such phenomena as turbulence, friction, cavitation are accompanied by non-Gaussian oscillations; crack sound emission, shock waves and other oscillations of the solution type and self-excited oscillations of mechanical and electric-mechanical systems are not the Gaussian signals either. Here the statistical dependence between frequency components of the signals usually exists, the fact which is ignored by traditional spectrum analysis; the measured power spectrum does not allow to distinguish such signals from Gaussian noise background and one from another. There is a problem of extending the range of measured by spectrum analyzer characteristics in a way that would enhance the research capabilities of nonlinear systems oscillations analyzing while the analyzer would keep its advantageous features such as high speed operation, compactness, cheapness. In the present article the principles, architecture and some other details of spectrum analyzer implementation, which partly solves the problem, are discussed.

2. Envelopes and Polyspectra.

The "classic" approach which enables in some cases to gain additional information about nonlinear system oscillations is to analyze the signal subjected to some preliminary nonlinear transformation, usually it comes to envelope extraction of frequency band limited signal

(frequency, amplitude, or power demodulation). The envelope spectrum analysis allows to detect cyclic variations of the noise level and of its spectral density maximum position. That is useful in some particular cases (for example in roller bearings diagnostics). Some mutual characteristics of the envelope and the initial signal useful for analyzing of wide range of effects in nonlinear mechanical systems will be construct below. Other, more general approach to nonlinear systems oscillations, investigation, is the measurement of so called "polyspectra" or "spectral moments". The n -order spectral moment of the signal $s(t)$ is denoted as $M(f_1, f_2, \dots, f_n)$ and is defined as follows.

Let $\{f_i\}$, $i=1, 2, \dots, n$ be some frequency set.

Let $s'(f_i; t)$ denote the frequency component of signal $s(t)$, being concentrated in some narrow band around the frequency f_i .

Let $\{d_i\}$, $d_i \in \{0, 1\}$, $i = 1, 2, \dots, n$ be some set of 0'es and 1'es.

Let symbols $c(0, z)$ and $c(1, z)$ be defined as follows $c(0, z) = z$, $c(1, z) = z^*$, $z \in C$, z^* is conjugate number to z .

The spectral product $P_n(f_1, f_2, \dots, f_n; t)$ is defined by the following equation

$$P_n(f_1, f_2, \dots, f_n; t) = c(d_1, s'(f_1; t) \otimes \dots$$

$$\bullet c(d_n, s'(f_n; t)) \quad (1')$$

In this article the negative frequency components of the signal are omitted in discussion. So we can simplify the definition of spectral product as follows.

Let $s(f; t)$ denote the signal $s(t)$ component of frequency $|f|$, being conjugated or not in accordance with sign of f :

$$s(f; t) = \begin{cases} s'(f; t), & \text{if } f > 0, \\ s'(-f; t), & \text{if } f < 0 \end{cases}$$

Then the definition (1') takes the form of $P_n(f_1, f_2, \dots, f_n; t) = s(f_1; t) \otimes s(f_2; t) \otimes \dots \otimes s(f_n; t)$ (1)

Now we can define the spectral moment

$M_n(f_1, f_2, \dots, f_n)$ as mean value of the spectral product $P_n(f_1, f_2, \dots, f_n; t)$.

$$M_n(f_1, f_2, \dots, f_n) = M(P_n(f_1, f_2, \dots, f_n; t)) \quad (2)$$

The spectral moment $M_n(f_1, f_2, \dots, f_n)$ of the stationary signal may differ from zero only on condition that

$$\sum_{i=1}^n f_i = 0. \quad (3)$$

So if the condition (3) is satisfied then n -order spectral moment comes to be the function of $n-1$ independent variables. In particular, the second order spectral moment depends only on one frequency and coincides with power spectrum. The spectrum moments of higher than second order are called polyspectra (the third order spectrum is called bispectrum and the fourth

order is called trispectrum). Mutual spectral moment of several signals is defined in the same way the only difference is that the spectral product contains frequency components of different signals. For full information of polyspectra and their properties see [1], [2], [3]. Here two most important at the moment properties are formulated.

Definition. Frequency set (f_1, f_2, \dots, f_n) is called degenerated set if it includes such subset (g_1, g_2, \dots, g_k) , $g \in \{f_j\}$, $k < n$ that

$$\sum_{i=1}^k g_i = 0.$$

Otherwise it is called non-degenerated set.

Statement 1.

Gaussian signal polyspectra are equal to zero upon non-degenerated frequency sets.

Statement 2.

Polyspectrum of the sum of statistically independent (non-correlated in every degree) signals is equal to the sum of polyspectra of the summands upon non-degenerated frequency sets.

In the works [1], [2], [3] Statements 1 and 2 are proved for spectral densities. Being slightly modified the statements keep true for polyspectra obtained from frequency components of finite band. The modification consist of two additional requirements:

- non-degenerated frequency sets must not be close-to-degenerated ones; frequency set (f_1, f_2, \dots, f_k) is called close-to-degenerated, if it includes such subset (g_1, g_2, \dots, g_k) that

$$|\sum_{i=1}^k g_i| < W/2, \quad k < n,$$

where W denotes the sum of bandwidths of filters separating components around the frequencies $|f_1|, |f_2|, \dots, |f_n|$; the filters must to have sufficient rejection apart of the band, in the Statement 2 the same filter is used for frequency component separating from the sum and from the summands.

In majority of known works devoted to non Gaussian signal investigations by means of poly-spectra only bispectrum was measured. It was measured on two-dimensional array of frequency pairs $(f_1, -f_2)$, the third frequency taken equal to $(f_1 - f_2)$ to satisfy the condition (1).

These works have shown the advantages of bispectrum estimation as the additive Gaussian noise component is suppressed, the oscillation modes phase relationship could be found, and using the coherence the oscillation modes interaction can be detected and interaction factors measured, etc.. But some obstacles both of principle and technical nature have emerged. We should remember that polyspectrum is the mean value of product, and the relation (mean value)/deviation decreases when the number of multiplicands increases, consequently the time required for averaging also increases. The result of the

measurement is the function of two variables, and it is more difficult to represent graphically and interpret than the power spectrum, depending only on one frequency. By the same reason much more calculations and memory space are required. For more than third order polyspectra all these difficulties grow rapidly. Comparing the researchers' requirements and our capabilities we have come to understand that the strategy "measure upon all frequency sets" should be abandoned, and we have to limit ourselves by the applications where the measurement of polyspectra upon separate points, lines and two dimensional arrays is required. We succeeded in two such applications, discussed further.

3. Connection between noise and its energy source.

The first application is relevant to the detection of random oscillation of higher frequency (noise) accompanying comparatively low frequency oscillation mode, and to the measurement of interaction parameters. Such noise is the consequence of energy dissipation in mechanical system with turbulence, friction, cavitation and crack propagation. The generated noise power, as a rule correlates with the square of low frequency component corresponding to its energy source. The other noise components do not depend statistically on this low frequency component. So by separating some frequency band from the initial signal and measuring bicoherence between the initial signal and its power-envelope (instantaneous power) in this band the connection between low frequency vibration and high frequency noise could be found. Making such measurements of envelopes over a number of frequency bands we can estimate the scale of inhomogeneity receiving the energy. In this case the bispectrum is measured very easily as the measurement should be performed only upon the line consisting of the sets of $(f, f, -2f)$ kind. But first some other signals are to be calculated on the basis of initial signals. Namely the vibration frequency band and noise frequency bands should be separated, the noise envelopes calculated, the low frequency part of envelopes separated (the latter operation is required to equalize sample rates of vibrations and envelopes). The band separation (so called "zoom") and envelope extraction operations themselves are performed by "classic" spectrum analyzers; only the higher flexibility and the feasibility of various versions should be provided. This problem will be discussed again in p.5 below.

4. Combinations of flows of constant form impulses.

Another application of polyspectra is relevant to the investigation of complex signal, which can be considered as a sum of Gaussian noise and several non-Gaussian components of special class, namely the flows of constant waveform impulses or polyharmonic quasi-periodical oscillations modulated in a random and slow way in the frequency and amplitude. It is assumed that the waveforms are different in different flows. It is also assumed that the cross-correlation (of every degrees) between impulse flows is equal to zero. The latter requirement is satisfied, in particular, if the flows are statistically independent and if different waveforms cannot arise simultaneously. Quasi-periodical summands are assumed to be of constant waveform different for different summands, cross-correlation between summands is assumed to be equal to zero. The latter requirement is satisfied, in particular, if the summands are independently frequency modulated or if their instantaneous frequencies never coincide. Instantaneous frequency distribution laws may coincide. The signal satisfying the proposed model will be called polycyclic. Polycyclic signals arise in consequence of repeated shocks, engine and pump operation and so on; electrical signals in muscles and nerves of animals are also close to the model. It appears that by the measurement and processing of trispectra the hypothesis about the observed signal is polycyclic could be examined; also the number of impulse flows could be determined and some general information about their waveforms gained; also it is possible to find out the number of quasi-periodical summands for which the instantaneous frequency distribution density differ from zero at arbitrary point h , and to gain some general information on their waveforms. If the measurement of 5-th or 6-th order polyspectra could be performed successfully (it requires a very long time of averaging) the waveform for each impulse flow or quasi-periodical component could be found. All these operations require rather complex processing of polyspectra.

5. Architecture, programming and control of analyzer.

The required flexibility of processing is provided on account of the equipment modular structure and the adequate software. Analyzer is relied on the virtual input channel concept which comes down to the following. In the course of reception of initial (primary) signals the secondary signals are computed, if required, on their basis; on the basis of primary and secondary signals the signals of third level are computed and so on. So the operation of cascaded signal transformations forming the virtual input channel is simulated. At user's disposal there is a library of transformations so that the desired virtual input channel could be organized by assigning the types of transforms and their interconnection. Numerical parameters of transformations could be specified in the course of programming, or just before measurement, and some of them during the measurement. The library contains the

transformations dealing with frequency transfer, filtering with decimation, filtering with compensation of time scale distortion, extracting amplitude and frequency envelopes, "weighted" summing of a number of signals. For analyzer the time is divided in the sequence of segments; the sequence of signal sampling in each segment along with logical validity flags forms a record. The sequence of records represents signal within the analyzer, it is the subject of transformations. The library contains the transformations generating flags on the basis of numerical values and users marks (range overload detector, external and hand triggering, etc.). Users marks are connected with current displayed record. The higher flexibility of the analyzer is attained due to the concept of user programmable statistic processing blocks. One or more records are used to form "time window". The analyzer allows to choose the time window selecting mode. The formed windows are then used for the calculation and accumulation of the following statistics:

- auto and cross energy spectra;
- histograms.
- polyspectra;

The user can specify frequency lists for polyspectra calculations (the list should not contain too many elements).

The analyzer supports a number of parallel blocks of statistic processing. The signals being processed in a block have to be sampled at the same rate.

To describe the virtual input channel, statistic processing blocks and output mode the special language is used. Before the beginning of measurement the analyzer adjusts itself according to the description, and a "virtual front panel" on the screen is generated. Then analyzer goes to dialog mode. In many cases the initial description is quite sufficient to perform the measurement upon two input signals. It describes the measuring circuit with a large number of logical flags, enabling to change the commutation of the virtual input channel and statistic processing blocks.

Concurrently with the measurement the analyzer displayed and puts out to external devices specified signals and current states of statistics. When measurement is accomplished the analyzer returns to dialogue mode, retaining the statistics as a result. The data stored during the measurement can be deleted or retained as files. The measurement results also could be stored in files. The stored signals and results could be repeatedly entered into the analyzer or processed by computer.

6. Coprocessors and system software.

The process of complex signals analyzing can be realized by some set of macrooperations. The signals to be analyzed, are entered into the system via several channels filling the data records. Such operations as filtration, extraction of envelopes, jumping Fourier transform use their input records and fill their output ones. Other operations, such as statistics calculation, use their input records and modify arrays of averaged data. When no operation requires the data, the record is dislocated and may be filled with new data. The throughput of general purpose microprocessor is not adequate for signal processing in real time, so to solve the problem the processors of different types have to be combined.

Input Processor Unit (IPU).

One of the coprocessors, analog and digital input processor performs the specified amplification and low frequency filtering of analogue signal, sampling and digitization, digital heterodyning, filtering and decimation. The coprocessor comprises multichannel low pass filters, scaling amplifiers, commutator, 16-bit A/D converter (ADC), digital heterodyne and readjustable digital filters.

Let F denote the input signal sampling rate, N is the number of channels. In each channel the passband of input analogue lowpass filter is $F/2.5$.

The possibility is provided to cut out narrow sections of wideband signals for further processing. To do so the specified spectrum band is transferred into the neighborhood of the zero frequency and the required bandwidth is filtered out by digital filters. The cascade of sequentially connected non-recursive digital low-pass filters with double decimation at each stage has proved to be efficient. The filter stage passes a signal in the band $[F_i/6; F_i/6]$, where F_i is the sampling frequency of the input sequence of the digital filter stage, and suppresses a signal at the level of -96 dB out of the band $[-F_i/3; F_i/3]$. After decimation the signal remains undistorted in the band

$[-F_o/3; F_o/3]$, where F_o is the sampling rate of the output sequence of the filter stage, $F_o = F_i/2$. The remaining part of frequency band is partially attenuated and aliased. Data sequences generated by digital filters are stored in the buffer memory for further processing. The throughput of IPU enables the signal processing with $F = 100$ kHz, $N=2$, i. e. it works in real time with 2 channels in the frequency bound of 40 kHz.

The modular hard- and software system possesses the unique capabilities of signal analyzing. There are several system configurations possible; minimal configuration hardware consists of only one Signal Processing Unit (SPU) connected with personal computer through the sequential or parallel or special interfaces.

Several, from two up to sixteen analogue input units of SPU can provide the spectrum analysis within the dynamic range of 96 dB with nominal signal amplitudes levels from 7.5 mV to 2 V, and frequency range of 0.01 to 20000 Hz. Several, up to four input units can be connected additionally which can provide the frequency range 0.01Hz to 100kHz.

The accessory interfaces allow to exchange data in accordance with IEEE-488 and byte parallel standards, and may be used for input or output of signal in digital form. The accessory complex-wave generator and analog input units are synchronized by the same clock frequency and may be used for reflexometrical measurements.

A serial number of unique samples of instruments and systems has been created on the base of development of the complexes which can evaluate the above mentioned high order spectra for analyzing hydroacoustic and hydrophysical signals or electromagnetic signals of navy objects. Several modes of the analyzers architecture are accomplish in our Institute: full hardware with build-in microcomputer, external device interfaced with PC and base on 16-bit Analog Devices

δ - Σ codecTM environment mount in PC which works in DOS or Windows TM mode. These complexes were used for development the testing environment for navy polygons and may be use in navy system of the barrier defense.

Reference.

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