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Measurement and Modeling of Probability Density Function in Coastal Range

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The results reported in this paper are directed toward the development of a statistical model of ambient noise at shallow water in coastal range. The data sets of ambient noise are analyzed for frequency band below 150 Hz, where distant shipping and wind-generated noises are observed in underwater environment. For three classes of sea state the PDFs of data sets have been modeled using the Gaussian - Gaussian mixture model. The method of moments has been applied for calculation of parameters of this model. The data sets were tested for randomness, independence and homogeneity. The estimated PDFs of ambient noise are nearly Gaussian. For 1-2 B sea state the PDF can be modeled as the Gaussian - Gaussian mixture.

1. Introduction

While there exist an extensive literature on the many aspects of ambient noise in the sea or ocean, investigations on the PDFs of ambient noise are relatively fewer. Dyer and co-workers have shown that one can go far in explaining observed fluctuations of ambient noise, by taking into account multipath interferences between the different arrivals at the measuring system. Having developed the statistics of decibel quantities, the authors showed that when several noise sources are present the statistics obey chi-square rather than log-normal distributions [7].

The PDFs of wind-generated, distant ship and snapping shrimp noises were analyzed by Bouvet and Schwartz [1]. These authors report that the PDF for distant ship noise obeys a Gaussian-Gaussian mixture distribution. The PDF of wind-generated noise obeys a Gaussian distribution and snapping shrimp noise is nonstationary.

The results quoted above involved a statistical analysis of ambient noise registered in depth sea. The primary objective of the present paper is to analyze of the PDF of ambient noise observed in the shallow water in costal range where, two main sources are being generated underwater noise: wind and distant ships.

2. The data collection

The data sets studied here are typical underwater acoustic noises and have been recorded with a single omnidirectional hydrophone. The sets of data have been recorded in costal range of South Baltic Sea. Noise observations were made in variety of hydrological conditions. The hydrophone was submerged to 20 meters below the sea surface and was mounted on a stationary measurement system. The ambient noise output of hydrophone was recorded in analog form on magnetic tape. Each sample was 60 sec long. The ambient noise sets of data have been sampled at a rate 512 Hz. Three sets of data have been processed for three different sea states: 1-2B, 3-4B and 5-6B.

Before doing a more detailed statistical investigation of data sets, the study of some very rough statistics can often be helpful. One-thousand samples of the data have been used. The data have been normalized to second order, i.e. their empirical global means and variances an the whole window are respectively 0 and 1. Fig. 1-3 shows the means for three classes of ambient noise corresponding to sea states: 1-2B, 3-4B and 5-6B. The means have been calculated based on 20 samples.

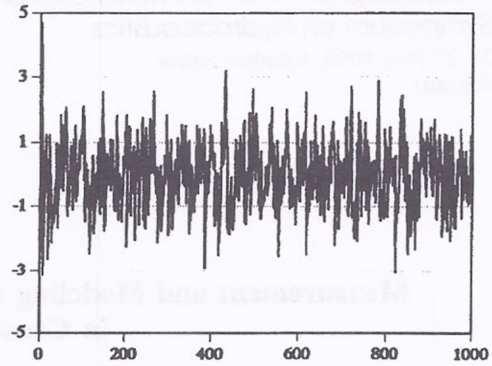

Fig. 1. The mean of 20 samples of ambient noise for sea state 1-2 B

3. Noise PDF modeling

The Gaussian - Gaussian mixture noise model have been developed in order to generalize the form of the ambient noise PDF. Mixture noise model have been chosen because of the nature of observed noise sources. Gaussian - Gaussian mixture noise model has a PDF composed of the sum of two Gaussian PDFs

$$p(x) = (1-\epsilon)g(x,\sigma_1^2) + \epsilon g(x,\sigma_2^2) \tag{1}$$

where:
ϵ - coefficient,
σ^2 - variance.

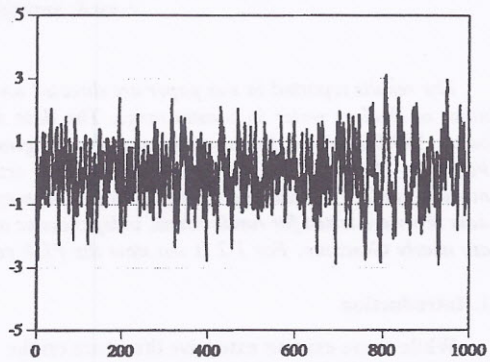

Fig. 2. The mean of 20 samples of ambient noise for sea state 3-4 B

Assuming PDF modeled by (1), with zero mean and unit variance, a straightforward method of moment estimation permits quite easily the calculation of the three relevant parameters. Since the variance is one, and denoting e_4 and e_6 the fourth and sixth moments of the observation, we have

$$(1-\epsilon)\sigma_1^2 + \epsilon\sigma_2^2 = 1 \tag{2}$$

$$(1-\epsilon)\sigma_1^4 + \epsilon\sigma_2^4 = e_4/3 \tag{3}$$

$$(1-\epsilon)\sigma_1^6 + \epsilon\sigma_2^6 = e_6/15 \tag{4}$$

As the exact solution of this system is complex, and as it is possible to assume that $\epsilon << 1$ and $e_4 < 3$, we can get these parameters approximately in a straightforward manner

$$\sigma_1^2 = \sqrt{e_4/3} \tag{5}$$

$$\sigma_2^2 = \left[e_6/15(1-\sqrt{e_4/3})\right]^{1/2} \tag{6}$$

$$\epsilon = \sqrt{e_6/15(1-\sqrt{e_4/3})^{3/2}} \tag{7}$$

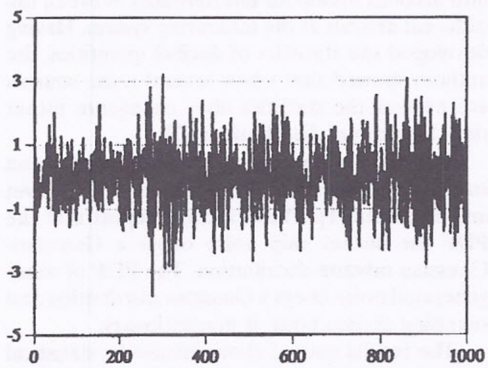

Fig. 3. The mean of 20 samples of ambient noise for

4. Statistical characterization of the data

Each of the data sets have been performed for randomness, independence and homogeneity. The runs up and down test for randomness, the Kolgomorov-Smirnov test for homogeneity have been applied to the data sets. In addition, the Kendall rank correlation test for independence have been applied.

For each of the data sets some empirical statistical measures have been computed. These statistical measures are the mean, the variance, the skewness and the kurtosis. As stated previously, the data sets were normalized, zero mean and unit variance.The statistical parameters of the data sets are shown in Table 1.

Table 1. The statistical parameters of the data sets

Sea state	Mean	Variance	Skewness	Kurtosis
1-2B	0.0032	1.0723	0.0846	2.6320
3-4B	-0.0015	1.0402	-0.0406	2.9791
5-6B	-0.0056	0.9891	-0.0138	2.8953

The quantiles of the empirical distribution function with respect to the Gaussian PDF are shown on Fig. 4 - 6. The curves seem to be fsymmetric, except for 1-2 B sea state, that indicates odd moments (skewness) greater than normal.

5. Probability density function modeling

The PDFs of data sets for three sea states have been modeled using the Gaussian - Gaussian mixture model described in paragraph 3. The method of moments have been applied for calculation of parameters of the Gaussian - Gaussian mixture model using formulas (5-7). The parameters of the Gaussian - Gaussian mixture model of ambient noise are shown in Table 2.

Table 2. The parameters of the Gaussian - Gaussian mixture model of ambient noise

Sea state	σ_1	σ_2	ϵ
1-2 B	0.98	0.14	0.001
3-4 B	1.01	-	-
5-6 B	1.01	-	-

For each data sets probability density functions were estimated using frequency method. Distribution fitting plots are shown on Fig. 6 - 9. The results of estimation are compared with a curve for Gaussian distribution.

6. Discussion

The results reported here are directed toward the development of a statistical model of ambient

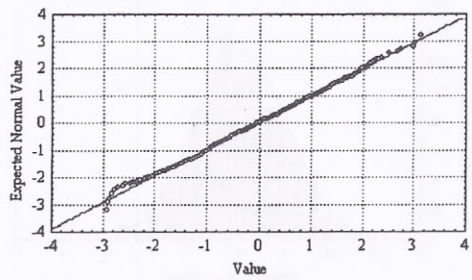

Fig. 4. Quantiles of ambient noise for sea state 1-2B

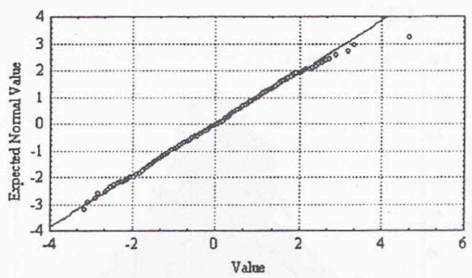

Fig. 5. Quantiles of ambient noise for sea state 3-4B

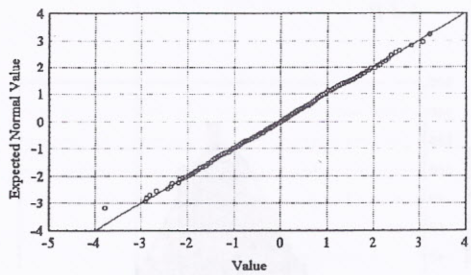

Fig. 6. Quantiles of ambient noise for sea state 5-6B

noise at shallow water in coastal range.

The data sets of ambient noise were analyzed for frequency band below 150 Hz, where distant shipping and wind-generated noises are observed in underwater environment. It caused, that Gaussian-Gaussian mixture model of PDF is physically motivated.

The statistical parameters of the data sets for sea state of 3-4 B and 5-6 B are closer to Gaussian distribution. The skewness is closer to zero and kurtossis is closer to the Gaussian value of 3 (Table 1). For 1-2 B sea state, the skewness is a bit more significant and the kurtossis is smaller to Gaussian value of 3.

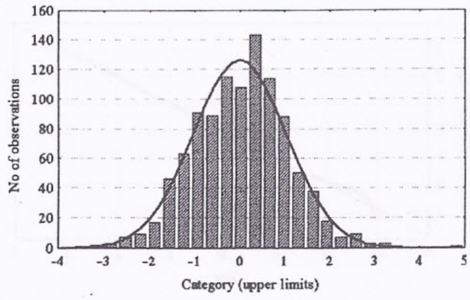

Fig. 7. The PDF of ambient noise for sea state of 1-2 B

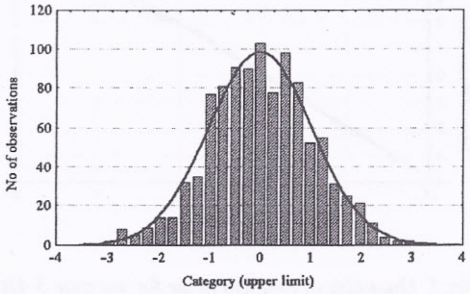

Fig. 8. The PDF of ambient noise for sea state of 3-4 B

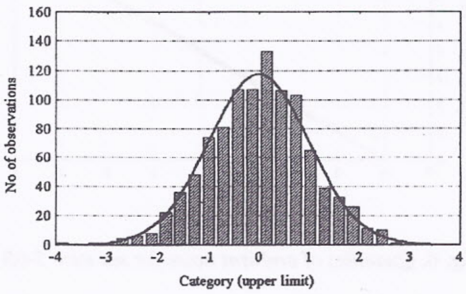

Fig. 9. The PDF of ambient noise for sea state of 5-6 B

The quantiles curves seem to be symmetric, except for the 1-2 B sea state, that indicates odd moments (skewness) greater than normal.

A look at the parameters for the Gaussian-Gaussian mixture model shows, that ambient noise data can be fitted by this model only for 1-2 B sea state. It is physically motivated, because the distant shipping noise is predominant in underwater environment for lower sea states (speed of wind < 10 m/s). However, the parameters obtained for the Gaussian-Gaussian model are little distinctive. Described results are similar to results published by Bouvet [1]. He analyzed the measurements of the ambient noise, which were carried out at deep water. The parameters of the Gaussian-Gaussian model for depht water are more distinctive than the parameters for shallow water. The Gaussian-Gaussian model can not be applied for sea states 3-4 B and 5-6 B (the σ_1 parameter of model is greater than 1, what caused that the expression under square in (6) is negative). This can be explained by fact, that for great speed of wind (greater than 10 m/s), the wind-generated noise is predominat in underwater environment.

At conclusion is worth to notice, that underwater environment at shallow water has its own specific, where a surface and bottom reverberation are very strong and multipath interferences between different arrivals at the receiver are observed.

The results of the above analysis are very useful for analyses of measurements of underwater noises at shallow water and signal detection. Future studies are recommended to explore the distribution in other frequency bands, and to identify the underlying ambient noise mechanisms.

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