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A Resonance Approach for the Plane-Wave Reflection Loss Model Including Sediment and Basement Rigidity

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The exact expression for reflection coefficient has been obtained with the Thomson-Haskell technique for the bottom model consisting of an elastic homogeneous layer overlying elastic half-space. The behaviour of the frequency-angular dependence for exact values of the reflection coefficient was studied. Inspection of exact expression for reflection coefficient shows that resonance behaviour will exhibit when the real part of denominator vanishes. The characteristic equations were obtained and roots of equations were found. Resonance positions for compressional and shear wave velocities were determined. Frequency-angular resonance positions were obtained both using resonance theory and exact computations. Comparison of obtained resonance positions was performed. The changes in the position for the resonance peaks for frequency- angular resonances as a function of the bottom model parameters were analysed. *This work was supported by the Russian Foundation for Basic Research (No.97-05-64712).*

1. Introduction

The usefulness of a resonance analysis for acoustic waves interacting with liquid layer and elastic plates embedded in a fluid have been illustrates in [1-2]. Exact expression for the resonance positions, widths, and strengths in the transmission and reflection coefficients have been written explicitly in terms of material and geometrical properties of the layer under consideration. The resonance theory of a fluid layer including viscous effects has been extended by R. Fiorito, W.Madigosky H.Uberall in [3], by the introduction of a complex sound velocity. In recent paper of R.Keltie [4] an analytical model of a compliant elastic coating attached to a submerged thin plate has been developed. The effects of incidence angle, frequency, location throughout the coating, and coating properties on the signal response are evaluated. An ultrasonic method for determination of the complete set of acoustical and geometrical properties of an isotropic layer embedded between two known materials at two angles is described by A.Lavrentyev and S.Rokhlin in [5].

In the present paper the resonance formalism, previously developed in [1-3] for an liquid layer (elastic plate) between two liquids, was extended for the case elastic layer covering an elastic half-space. Viscosity (absorptive effects) were took into account by the introduction of a complex velocities, the imaginary part of which is written in terms of an absorption loss factor. Exact analytical expressions for the reflection coefficient have been derived using Thomson-Haskell technique. Inspection of exact equations for the reflection coefficient shows that resonance behaviour will exhibit when the real part of the reflection coefficient denominator vanishes. The characteristic equations were obtained and roots of equations were found. Analytical expressions determined resonance positions connected with compressional and shear wave velocities were obtained. Comparison of the resonance approach and exact computation was performed. Changes of resonance peaks positions as a function of the bottom model parameters were analysed on the frequency-angular plane.

2. Model Formulation

The physical model used in the study of frequency-angular resonance is shown in Fig.1; the quantities c_0, ρ_0 are the sound speed and the density of a liquid half-space, d, c_l, c_t, ρ are the thickness, compressional and shear wave velocities, density of the layer, $c_{l\infty}$, $c_{t\infty}$, ρ_∞ are compressional and shear wave velocities, density of the elastic half-space. Sediment parameters are permitted to be constant within elastic layer. The water column and elastic

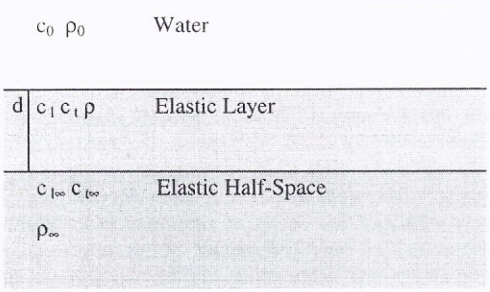

Fig. 1. Geometry of the seabed model.

half-space (substrate) are assumed to be homogeneous and semi-infinite. In the layer, including the substrate, the effects of the attenuation are taken into account by assuming shear (c_t) and compressional (c_l) wave velocities are complex $c=c_r+ic_i$. It is requires complex wave numbers

$\kappa=\omega(1-i(c_i/c_r))/(1+i(c_i/c_r)^2)$, $\omega=2\pi f$, f is frequency. Attenuation within the water column will be neglected. It is assumed that the displacement fields $\mathbf{U}=\mathbf{U}(U_x,U_y=0,U_z)$ can be written in terms of the scalar φ and the vector potential ψ:

$$\mathbf{U}=\mathrm{grad}\varphi + \mathrm{rot}\psi = \nabla\varphi + \nabla\cdot\psi. \quad (1)$$

Components of the displacement in the layer for rectangular coordinates:

$$U_x=\partial\varphi/\partial x-\partial\psi/\partial z,\ U_z=\partial\varphi/\partial z+\partial\psi/\partial x, \quad (2)$$

where φ and ψ are governed by the Helmholtz equations:

$$\Delta\varphi + \alpha^2\varphi=0,\ \Delta\psi+ \beta^2\psi=0, \quad (3)$$

$\alpha^2= \kappa_l^2-\xi^2$, $\beta^2= \kappa_t^2-\xi^2$, $\xi= \kappa \sin(\theta) = \kappa_l \sin(\theta_l) = \kappa_t \sin(\theta_t)$, θ - incident angle. In addition to components of the displacement given in Eq.(2) components of the stress tensor, which are continuous across an interface can be written in terms of φ and ψ potentials:

$$\sigma_{xz} =2\mu(\partial^2\varphi/\partial x\partial z-\partial^2\psi/\partial z^2),$$

$$\sigma_{zz}=-\lambda\partial^2\varphi/\partial x^2+(\lambda+2\mu)\partial^2\varphi/\partial z^2+\partial^2\psi/\partial x\partial z, \quad (4)$$

where λ and μ are Lame constants connected with compressional $c_l=\sqrt{(\lambda+2\mu)/\rho}$ and shear $c_t=\sqrt{\mu/\rho}$ wave velocities. Solutions of Eq.(3) may be written in terms of potentials, describing compressional and shear waves:

$$\varphi = \varphi^+ \exp(i\alpha z) + \varphi^- \exp(-i\alpha z),$$

$$\psi =\psi^+ \exp(i\beta z) + \psi^- \exp(-i\beta z), \quad (5)$$

For solution of the system of equations W.Thomson and N.Haskell [6,7] proposed the matrix method, in which the system of linear algebraic boundary equations are replaced by the matrix equation: $Z_0=DZ_\infty$, where $Z_\infty =[\varphi^+,\varphi_j,\psi^+,\psi^-]^T$ is the row vector, $L=[\exp(+i\delta),\ \exp(-i\delta),\ \exp(+i\eta),\ \exp(-i\eta)]^T$ is the diagonal matrix. Here the superscript T indicates the transpose of the row vector, $D=Q\ A_j^{-1}A_j\ L$ is the propagator matrix, A_j is the characteristic matrix of 4-th order for a layer, Q is the matrix of 2-nd order for liquid a half-space:

$$A_j=\begin{vmatrix} i\xi & i\xi & -i\beta & i\beta \\ i\alpha & -i\alpha & i\xi & i\xi \\ 2\mu\xi\gamma & 2\mu\xi\gamma & -2\mu\xi\beta & 2\mu\xi\beta \\ -2\mu\xi\alpha & 2\mu\xi\alpha & -2\mu\xi\gamma & -2\mu\xi\gamma \end{vmatrix} \quad (6)$$

$$Q=\begin{bmatrix} i\alpha_0 & -i\alpha_0 \\ -\omega^2\rho_0 & -\omega^2\rho_0 \end{bmatrix}$$

where $\gamma=\xi-k_t^2/2\xi$, $k_t=\omega/c_t$

The solutions of the system of matrix equations can be obtained by the Kramer's rule, $\chi_k = \Delta_k / \Delta$ (k=1,2,3), where χ_k are reflection and refraction coefficients, Δ is the main determinant of the system.

Exact expression for the complex reflection coefficient was obtained and may be written in the form

$$V=((A^2-B^2+C^2-D^2)^2+4(CB+DA)^2)/ \quad (7)$$

$$((A+B)^2+(C-D)^2)(A^2-B^2+C^2-D^2+2i(CB+DA)),$$

where functions A, B, C and D, contained in (7), connected both material parameters of media and angle-frequency-thickness variables $\delta=\alpha d$ and $\eta=\beta d$. Variables δ and η combined with positions of frequency-angular resonances for compressional and shear wave velocities. Expressions for the functions A, B, C and D are not reduced due to its awkwardness.

Note it is possible to obtain position of resonance's minima and maxima of the reflection coefficient by using the Ferma theorem for the analytical expression (7). However, equation obtained after differentiation of V can't be solved analytically.

3. Resonance Formalism

Inspection of the exact equation (7) for the reflection coefficient shows that resonance behaviour will exhibit when the real part of the denominator vanish, i.e., when the characteristic equations are satisfied:

$$(A+B)^2+(C-D)^2 = 0, \qquad (8)$$

or

$$A^2-B^2+C^2-D^2 = 0. \qquad (9)$$

If the real part of the denominator vanish the phase of V equal to $\pm\pi/2$.

The Fig.2 illustrates this fact. In the Fig.2 modulus and phase of reflection coefficient are shown in the form of isolines.

After variables separation the characteristic equation (8) may be written in the form:

$$K_1\cos(\delta)\cos(\eta)+K_2\cos(\delta)^2\cos(\eta)^2 + K_3\sin(\delta)\sin(\eta)+K_4\cos(\delta)^2+K_5\cos(\eta)^2 + K_6\cos(\delta)\cos(\eta)\sin(\delta)\sin(\eta)+K_7=0, \qquad (10)$$

where constants K_1, K_2, K_3, K_4, K_5, K_6, K_7 are connected only with material parameters of media.

In the case of the elastic layer covering elastic half-space the Eq.(10) may be solved separately for δ and η variables. Solutions will determine the positions of resonance for two types of waves at the frequency-angular plane. The solutions for δ and η may be obtained by solving of additional 4-th order equations:

$$(-K_1\cos(\eta) + K_2\cos(\eta)^2 + K_4 + K_5\cos(\eta)^2 + K_7)X^4 + (2K_3\sin(\eta) - 2K_6\cos(\eta)\sin(\eta))X^3 + (-2K_2\cos(\eta)^2 - 2K_4 + 2K_5\cos(\eta)^2 + 2K_7)X^2 + (2K_3\sin(\eta) + 2K_6\cos(\eta)\sin(\eta))X + K_1\cos(\eta)+K_2\cos(\eta)^2 + K_4 + K_5\cos(\eta)^2 + K_7 = 0, \qquad (11)$$

$$(-K_1\cos(\delta) + K_2\cos(\delta)^2 + K_4\cos(\delta)^2 + K_5 + K_7)X^4 + (2K_3\sin(\delta) - 2K_6\cos(\delta)\sin(\delta))X^3 + (-2K_2\cos(\delta)^2 + 2K_4\cos(\delta)^2 - 2K_5 + 2K_7)X^2 + (2K_3\sin(\delta) + 2K_6\cos(\delta)\sin(\delta))X + K_1\cos(\delta)+K_2\cos(\delta)^2 + K_4\cos(\delta)^2+K_5+K_7=0. \qquad (12)$$

Then δ and η may be found as

$$\delta = 2\arctan(X),$$

$$\eta = 2\arctan(X), \qquad (13)$$

$$\delta_n=\delta+2\pi n,$$

$$\eta_n=\eta+2\pi n,\ n=0,1,2,\ldots, \qquad (14)$$

X are roots of Eq.(11), Eq.(12) for δ and η respectively, n is the resonance number. These solutions will true in broad range of grazing angles θ.

Resonance expression for the reflection coefficient may be obtained as a sum of resonance terms, both in the frequency variable and in the angular variable. Mathematically, it is correspond to retaining only linear terms in the expansion of expression for V in the Taylor row around their resonance values δ_n .and η_n.

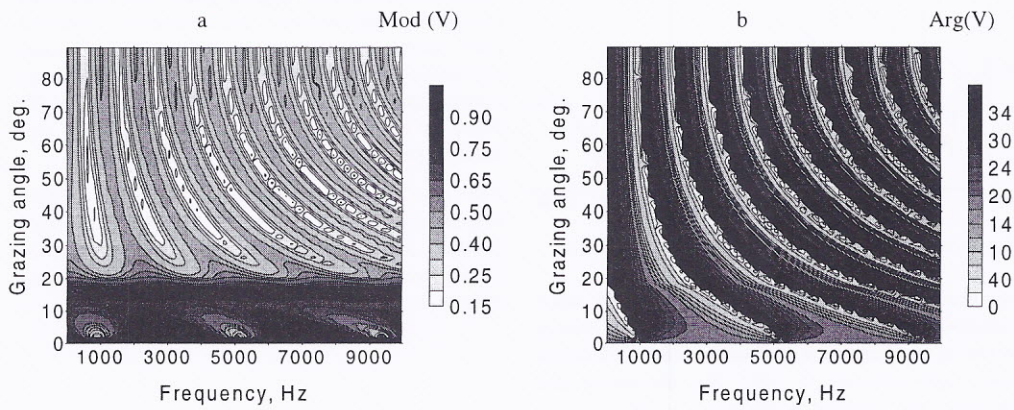

Fig.2 The modulus (a) and the phase (b) of the reflection coefficient

4. Frequency and Angular Resonances

Results of numerical simulation of the exact reflection coefficient (7) are shown in the Fig.3 on the frequency-grazing angle plane ($c_{l\infty}>c_0>c_l$). The plane wave reflection coefficient consist from regular sequence of minima and maxima connected with resonances. Resonances of the reflection coefficient are functions of frequency, grazing angle and seabed parameters. Note that frequency resonances may be observed at very low grazing angles $\theta \approx 1\text{-}2^{o}$ (Fig.3). It can result in increasing of sound propagation loss in the shallow sea.

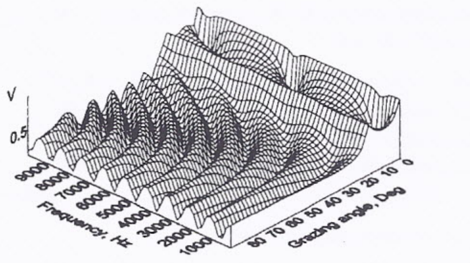

Fig.3. Modulus of reflection coefficient, plotted as a function of frequency and grazing angle.

The influence of absorption on resonance peaks of the reflection coefficient illustrate the Fig.4. Results of computation for a 0.7 m-thick sediment layer using the actual absorption coefficients in the layer and layer shear wave velocity ($\upsilon_l = 0.01$ dB/m, $\upsilon_t = 0.001$ dB/m, $c_t = 250$ m/s) presented for two grazing angles $\theta = 3^0$ and $\theta = 61^0$ (curves 1 and 2 respectively).

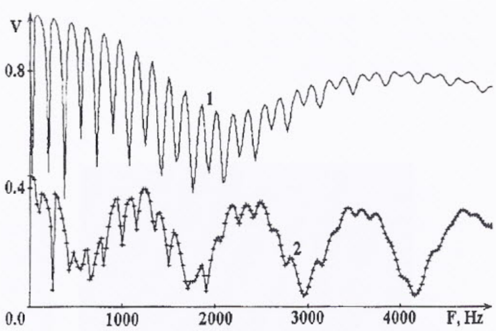

Fig. 4 Influence of absorption on resonance peaks

The absorption decrease the resonances peaks when frequency increase, due to the dissipation of energy in the layer. Frequency resonance width Γ decrease when angle increase ($\Gamma \sim 1/\theta$). The absorption also decrease the angular resonances peaks. Angular resonance width γ increase with increasing of grazing angle.

At normal incident angle there is not tangential components of compressional wave on the water-sediment boundary and shear wave can't be exited. So there is no influence of layer rigidity on resonance peaks at normal incident angle. Investigation of influence of sediment and substrate shear speeds on sound reflection coefficients at low grazing angles actual for study of sound propagation in shallow water. Numerical research of influence of sediment and substrate rigidity on the reflection coefficient resonances for fixed grazing angle $\theta = 1^0$ show that frequency displacement of resonance peaks are observed when the shear speed in the substrate $c_{t\infty}$ is varied (Fig. 5). This displacement principally may be used for reconstruction of the substrate parameters.

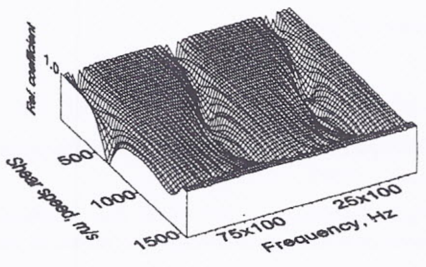

Fig.5. Dependence of resonance position from share speed in the substrate for fixed grazing angle $\theta = 1^o$.

5. Comparison of Resonance Approach and Exact Computations

Comparison of resonance approach and exact computations was performed for the elastic layer lying on the elastic substrate. The resonance positions on the frequency-angular plane were calculated using exact expression (7) and resonance expression (14) (n=0). Resonances associated with δ were calculated for three different c_l in the layer. Exact computation of resonance position shown on the Fig.6 as solid line and resonance approach calculation shown by markers.

The curve 1,2,3 correspond to c_l=1455 m/s, c_l=1520 m/s and c_l=1475 m/s respectively. Computation were fulfilled for the grazing angles low than critical. Dependence of resonance position from c_l is clearly observed.

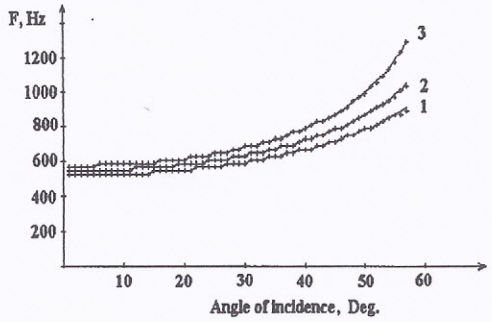

Fig. 6. Resonance position on angle-frequency plane for three different c_l. Solid line – exact computations, markers – computations using resonance formalism

6. Conclusions

The exact expression for the reflection coefficients have been obtained with the Thomson-Haskell technique. Viscous (absorptive) effects within the sediment and substrate were incorporated in these equations by introducing a complex compressional and shear wave velocities the imaginary part of which is written in terms of an absorption loss factor, in turn required complex wavenumbers. Inspection of the exact expression for reflection coefficient shows that resonance positions may be obtained when the real part of the denominator vanish, i.e. when the characteristic equations for the resonance positions are equal to zeros. The behaviour of the frequency-angular dependences for exact values of the reflection coefficients was studied. Influence of the effect of absorption and bottom characteristics influence on the resonance structure was analysed. Particular attention was devoted to low grazing angle resonances. The influence of the shear speed in the sediment and substrate were investigated. The resonance positions on the angle-frequency plane were calculated using exact expression and resonance expressions. Comparison of resonance positions obtained from the resonance approach and exact calculations was performed and good agreement was obtained. Dependence of resonance position from bottom model parameters was analysed.

7. Acknowledgements

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