MODELING OF SOUND PROPAGATION TRACT USING WALSH TRANSFORM

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Paper presents problem of modeling sound propagation tract basis on orthogonal system of Walsh functions. Proposed method allow for determination of sound propagation tract model which among others thing should gives possibility to prediction parameters of sound source. First of all the technique of Walsh Transform is presented in details. The mathematical description of Walsh Transform as well as detail description of calculation process were presented. Secondly the example results of preliminary research carried out on simulated signals as well as vibrations and hydroacoustics signals acquired during real condition measurement of motorboat were presented. At the end obtained results were discussed and direction of the future research were pointed.

INTRODUCTION

Issues of modeling is one of the fundamental problems in many areas of science. Knowledge of the exact mathematical description of the object provides many opportunities both at the stage of conducting research to develop new methods and algorithms through the design phase where it gives possibility to check the correctness of the proposed solutions and ending on practical applications where knowledge of object model allows for achievement better results. Having a model of sound propagation tract opens up new research possibilities and it can also be used in practice. Among other things, having such a model it is possible to predict how sound with assumed parameters will propagate on the way from the source to the point where it will be registered. Having possibility to reverse of such model and having an output signal it is possible to conclude about the input signal.

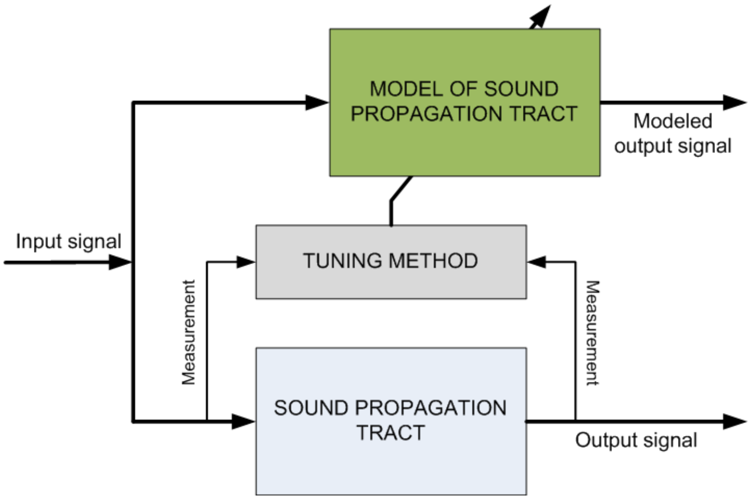

Fig. 1. The idea of modeling sound propagation tract

To solve the problem of modeling the sound propagation tract was decided to use the methods successfully used in the automation and robotics to identify objects of control. It is assumed that sound propagation tract will be treated as a black box of which there is absolutely no information. There will be also two signals called input signal and output signal. The input signal is recorded original signal which will be propagated in the tract and the output signal is the signal recorded after passing through the sound propagation tract. On the basis of the recorded those two signals it will be modeled the propagation tract. This idea is shown on figure 1. For example, considering the ship as the input signal can be treated vibration of the propulsion system recorded using accelerometers situated on the main engines constituting the sound source and the output signal will be hydroacoustic signature registered in a water area where the ship is located (figure 2).

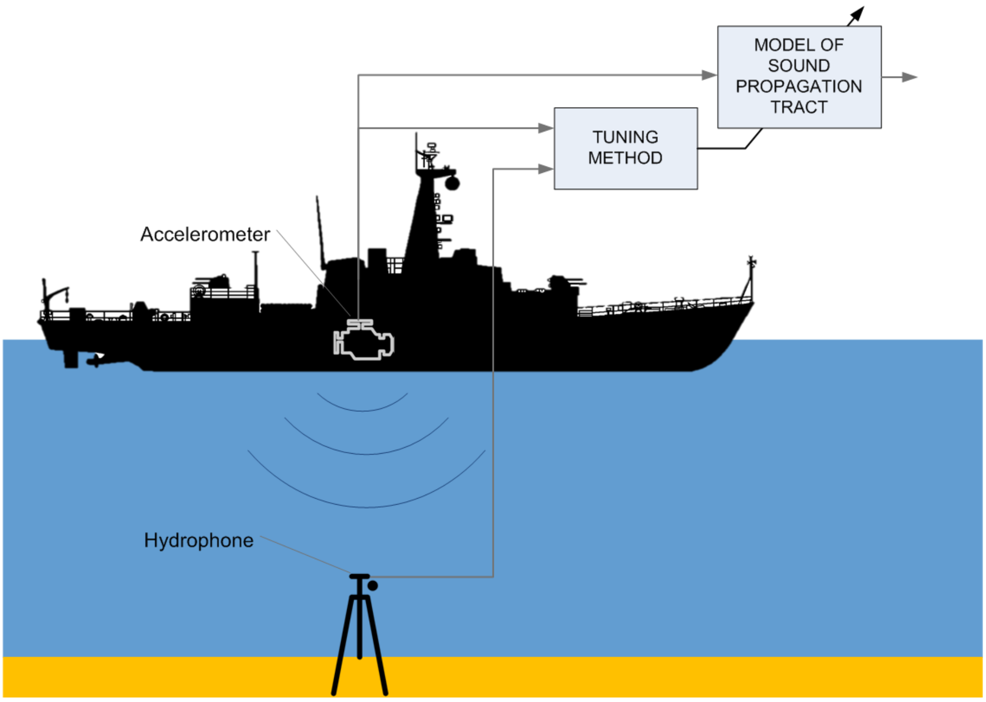

Fig. 2. Example of data used to model the sound propagation tract

For modeling it will be used Walsh Transform. Among the piecewise constant functions widest practical use have found the Walsh functions which binary nature has a particularly major meaning when digital technology is used to perform the task of identification. Orthogonal system of Walsh functions are used to identify parameters of dynamics model of dynamic objects. Walsh functions are widely used for the analysis and synthesis of dynamical systems described by linear and nonlinear differential and intergral equations. These issues are solved by using Walsh functions through the so-called operating matrices, which are used to transform the differential equation or integral to the corresponding algebraic equation.

1. WALSH TRANSFORM

Sound propagation tract will be considered as n-dimensional linear dynamic object described by the equation:

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

where: x_k – the output signal; u_k – the input signal; A – a state matrix; B – an input matrix.

The purpose of the identification is to determine the unknown coefficients of the A and B based on the measurement vector x and u.

Vectors x and u can be presented in the form of the orthogonal development in the terms of Walsh basis functions in the form of [4]:

$$x_k = Fw_k, u_k = Hw_k \tag{2}$$

where: k - index of discrete time; w_k - k-th vector of discrete values Walsh functions; F - matrix of coefficients of the orthogonal development of the output signal; H - matrix of coefficients of orthogonal development of the input signal.

After summing up both sides of equation (1) the following expression can be presented [4]:

$$\sum_{i=0}^{k} x_{i+1} = \sum_{i=0}^{k} (Ax_i + Bu_i) \tag{3}$$

By making simple transformations and substitutions, this equation is reduced to the form:

$$\sum_{i=0}^{k} Fw_i - Fw_{k+1} - z_0 = \sum_{i=0}^{k} AFw_i + \sum_{i=0}^{k} BHw_i \tag{4}$$

The initial state vector x_0 can be presented in terms of the development of orthogonal Walsh functions as follows [4]:

$$x_0 = [x_0, 0, 0, \dots, 0]w_k \tag{5}$$

Using the properties of Walsh functions, can be formulated the following equation [4]:

$$FSw_k + FZw_k - z_0w_k = AFSw_k + BHSw_k \tag{6}$$

where: S - operating matrix for the operation of aggregation of Walsh functions; Z - operational matrix for the operation of shift of Walsh functions.

Except for matrices A and B, which should be identified all other vectors and matrices in the equation (6) are known.

Assuming that all the matrix coefficients A and B are unknown number of unknowns is equal to $n(n+r)$ where: n - length of the output signal, r - length of the input signal.

Since the previous equation contains n equations, so to obtain unambiguous solution the set of equations should be extended, taking for example $n + r$ arbitrarily selected samples of Walsh functions [4].

The extended system of equations can be written as follows [4]:

$$\begin{gathered} FSw_{k_1} + FZw_{k_1} - z_0 w_{k_1} = AFSw_{k_1} + BHSw_{k_1} \\ FSw_{k_2} + FZw_{k_2} - z_0 w_{k_2} = AFSw_{k_2} + BHSw_{k_2} \\ \dots \\ FSw_{k_{n+r}} + FZw_{k_{n+r}} - z_0 w_{k_{n+r}} = AFSw_{k_{n+r}} + BHSw_{k_{n+r}} \end{gathered} \tag{7}$$

By substitution [4]:

$$\begin{gathered} T = FS + FZ - x_0 \\ W' = \left[w_{k_1}, w_{k_2}, \dots, w_{k_{n+r}}\right] \\ R = TW' \\ P = FSW' \\ Q = HSW' \end{gathered} \tag{8}$$

system of equations takes the form:

$$R = AB\,PQ \tag{9}$$

where: $AB = [A\ B]$, $PQ = \begin{bmatrix} P \\ Q \end{bmatrix}$

Assuming that: $det(PQ) \neq 0$ values of matrix AB can be determined according to [4]:

$$AB = R(PQ)^T[PQ(PQ)^T]^{-1} \tag{10}$$

Walsh functions forms a family of binary orthogonal functions belonging to the class of piecewise constant functions [1, 2]. It is convenient to define these functions by using the relationship with Rademacher functions, which are defined as [3]:

$$R_k(t) = \begin{cases} +1\ for\ \dfrac{i-1}{2^{k+1}} \leq t < \dfrac{i}{2^{k+1}}\ when\ i\ is\ odd \\ -1\ for\ for\ \dfrac{i-1}{2^{k+1}} \leq t < \dfrac{i}{2^{k+1}}\ when\ i\ is\ even \end{cases} \tag{11}$$

where: $\mathrm{k} = 0, 1, 2, \dots;\ \mathrm{i} = 1, 2, \dots, 2^{\mathrm{k}+1}$

The relationship between Walsh functions $w_n(t)$ and Rademacher functions $R_k(t)$ is as follows [3, 4]:

$$w_0(t) = 1 \; for \; 0 \le t < 1$$
$$w_n(t) = \prod_{i=0}^{n} \left(R_k(t)\right)^{n_i} \quad for \; 0 \le t < 1 \tag{12}$$

where: n_i - the value of i-th position after conversion of n to binary system.

Using the above definition the first eight Walsh functions can be represented as follows:

$$\begin{gathered}
w_0(t) = w_{000b}(t) = 1 \\
w_1(t) = w_{001b}(t) = R_0(t) \\
w_2(t) = w_{010b}(t) = R_1(t) \\
w_3(t) = w_{011b}(t) = R_0(t)R_1(t) \\
w_4(t) = W_{100b}(t) = R_2(t) \\
w_5(t) = w_{101b}(t) = R_2(t)R_0(t) \\
w_6(t) = w_{110b}(t) = R_2(t)R_1(t) \\
w_7(t) = w_{111b}(t) = R_2(t)R_1(t)R_0(t)
\end{gathered} \tag{13}$$

Walsh functions are conveniently presented in the form of a matrix called the matrix of Walsh. The matrix for the first eight Walsh functions is as follows [3]:

$$W = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{bmatrix} \tag{14}$$

With the Walsh functions are related so-called operation matrixes. During the identification process are using only two matrixes: aggregation operating and shift operating. The operating matrix for the aggregation operation is defined as follows [3]:

$$\sum_{i=0}^{k} w_i = S w_k \tag{15}$$

where: S - operating matrix for the aggregation operation; w_k - k-th column of Walsh matrix;

The S matrix for $N = 8$ is as follows:

$$S = \begin{bmatrix} 4.5 & -2 & -1 & 0 & -0.5 & 0 & 0 & 0 \\ 2 & 0.5 & 0 & -1 & 0 & -0.5 & 0 & 0 \\ 1 & 0 & 0.5 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & 0 & 0 & -0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{16}$$

The matrix for the shift operation is defined as [3]:

$$w_{k+1} = Zw_k \tag{17}$$

where: Z - operating matrix for shift operations; w_k - k-th column of Walsh matrix.

Z matrix for $N = 8$ has form as follows:

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0 & -0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -0.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0.5 & 0 & -0.5 & 0 & -0.5 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & -0.5 & 0 & 0.5 & 0 & -0.5 \end{bmatrix} \tag{18}$$

2. RESULTS OF RESEARCH

The presented method has been implemented in Matlab programing environment. In the first phase of research have been performed tests where the input signal was simulated. As input signal was used sinusoid of a given parameters (frequency, amplitude, and starting phase). It was assumed that the output signal is a signal consisting of the sum of sine waves at the same amplitude and starting phase as input signal and frequencies the same as input signal, multiplied by 2.6 and divided by 2.6. To the such formulated output signal was added random noise signal with amplitude of 1/10 of input single amplitude. After calculation basis on input and output signal was created model of sound propagation tract. The values of matrix coefficients A and B were presented in graphic form on figure 3. Created model was used to determine the modeled output signal in order to check the correctness of calculations. To the input of the model was given simulated input signal and the output signal from the model – modeled output signal was compared to the expected signal. The figure 3 shows the waveform of the simulated input signal (green color), simulated output signal (blue color) and modeled output signal obtained from model (red color). As it can be seen there is a great similarity of modeled output signal and to the expected one.

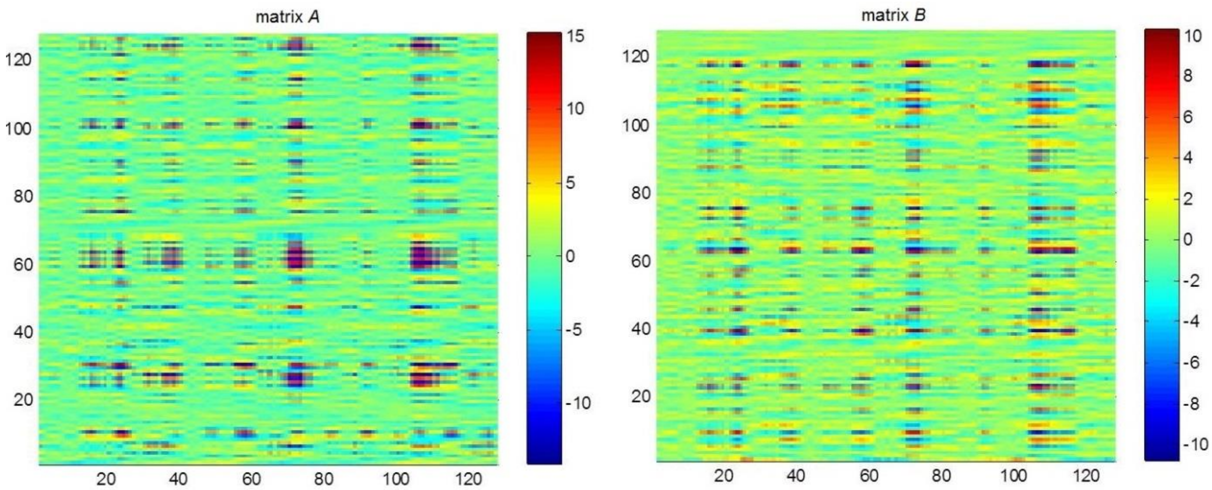

Fig. 3. Graphic presentation of calculated matrixes *A* and *B* of created model of sound propagation tract

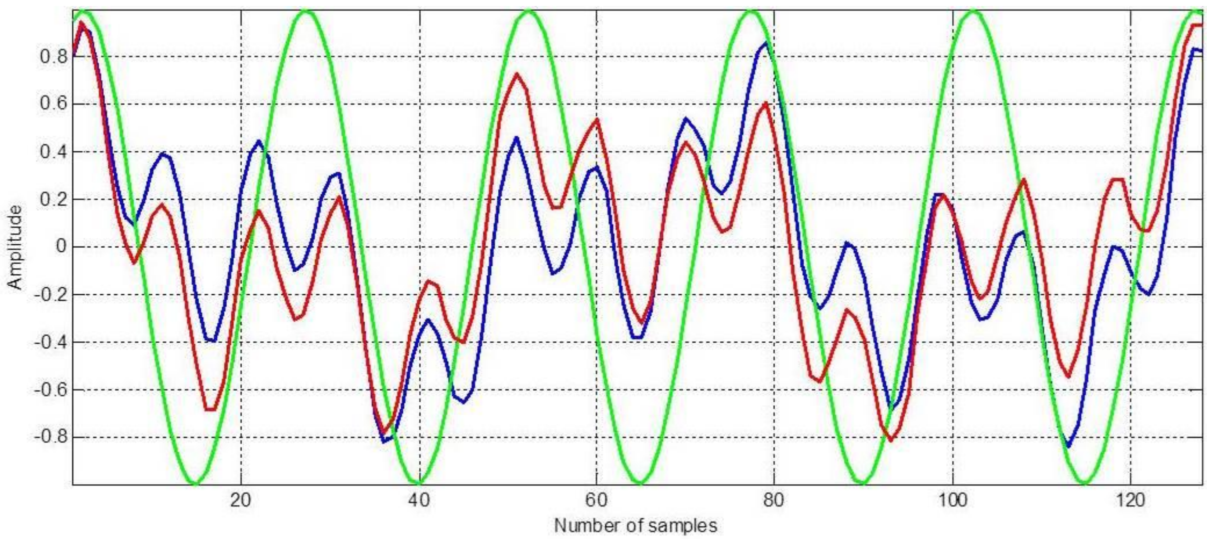

Fig. 4. Waveform of simulated and modeled signals; green color – simulated input signal, blue color – simulated output signal, red color – modeled output signal

In the next phase of researches, were used signals recorded during the test in real conditions. The study was done on motorboat driven by internal combustion engine. During measurement were recorded signals from an accelerometer mounted on the engine and hydrophone placed in water at about 5 meters from the side of the motorboat. Acquired signals waveforms and their amplitude spectra are shown on the figure 6. Basis on those signals has been created model of sound propagation tract using previously presented method. The calculated matrixes were graphically shown on figures 7. Next to the input of created model of sound propagation tract was given the part of recorded input signal and modeled output signal was compared with recorded output signal. The example part of input signal (green color) as well as original output signal (blue color) and modeled output signal (red color) were presented on figure 7. As it can be seen output signal from created model is very similar to the originally recorded signal.

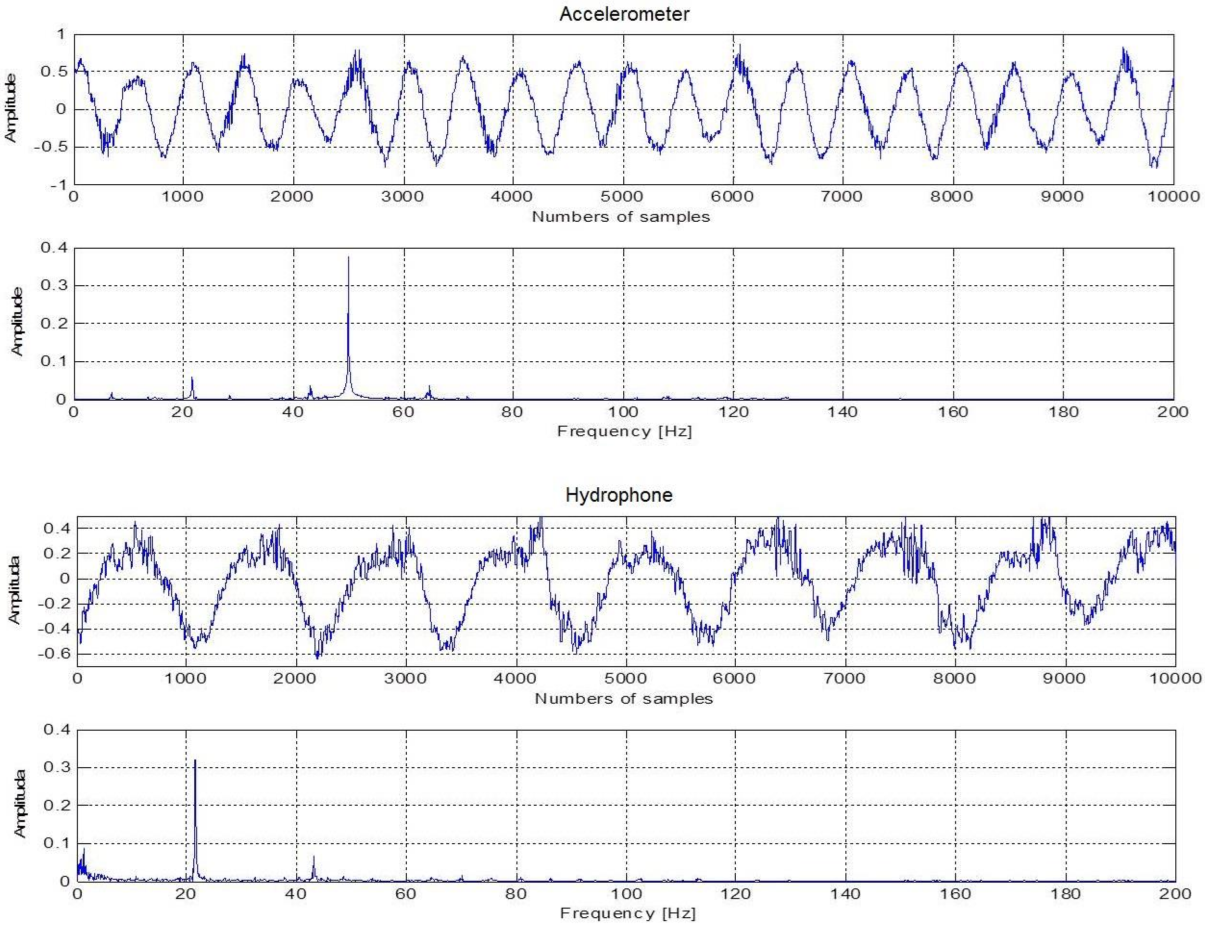

Fig. 5. Waveform and amplitude spectra of recorded during real condition measurement signals

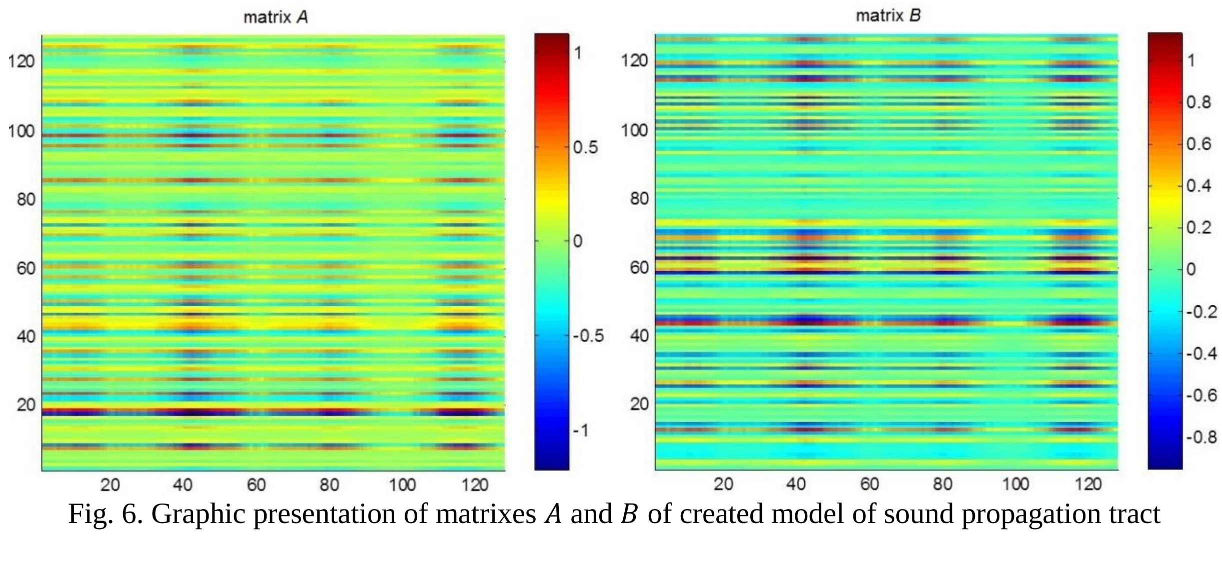

Fig. 6. Graphic presentation of matrixes A and B of created model of sound propagation tract

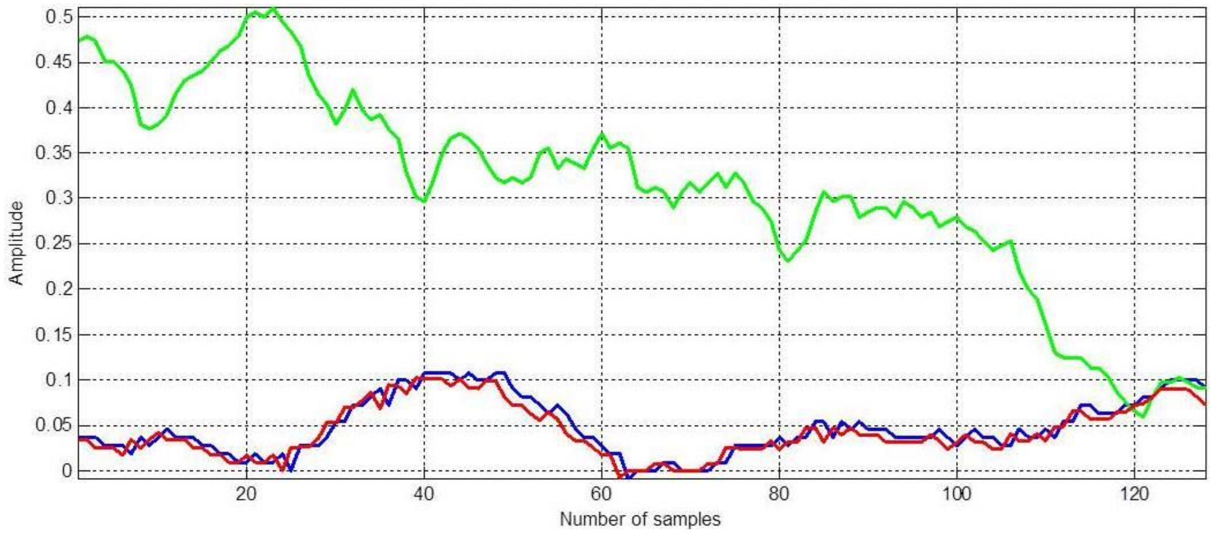

Fig. 7. Part of signals used and obtained in researches; green color – measured input signal, blue color – measured output signal, red color – modeled output signal

3. SUMMARY

Summarizing, after a first preliminary studies it can be concluded that the Walsh Transform is promising in terms of modeling sound propagation tract. Results obtained suggest a fairly high accuracy of the identified model.

Of course, there is still a lot of issues to resolve. An important part of the modeling process is to evaluate the model, so how to determine whether the model describes well the real phenomenon? The answer to this question is obviously not easy. The next question is: how to determine whether a used set of measurements is representative for all possible situations? So how well the model describes the data held outside the range of measurements? For this purpose should be carried out much more research in laboratory and real conditions. Further studies could also consider whether the created model of sound propagation tract subject to the law of superposition. Positive answer to this question opens the way to new possibilities of sound propagation analysis using models.

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