MULTIBEAM SONAR RECORDS DATA DECIMATION USING HIERARCHICAL SPLINE INTERPOLATION

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Multibeam sonar records feature high vertical and horizontal resolution. Interpolating and approximating and eventually displaying scattered 3D raster data of high volume leads to some difficulties related to a computer processing power. The paper presents some advantages of using hierarchical splines. The proposed approach consists of two stages: firstly, all acquired multibeam sonar raw data are interpolated with high density uniform spline interpolation. The knots and control points of interpolated network are saved for defined resolution level. In the next stage preprocessed high resolution data are combined with low resolution data sets following prior knot decimation process. Such approach allow real time 3D displaying of multibeam sonar data for different zoom levels.

INTRODUCTION

The bathymetric data features wide range of vertical and horizontal resolution, e.g. records from multibeam sonar (MBS) posses decimeter or even higher resolution and, on the other side ocean bathymetry features 1 km resolution so MBS records may feature, in many areas, obvious redundancy and ambiguity [2], but at the same time can be useful in an approximation approach. The paper presents the approximation approach using a hierarchical spline techniques. The approach allows for a flexible and appropriate resolution for different scales in the process of visualization and bottom imaging.

1. SPLINE FUNCTIONS

Spline functions can be expressed as linear combination:

$$s(x) = \sum_{k \in Z} c(k)\varphi^n(x-k) \tag{1}$$

where $c(k)$ is a control point, and φ^n a base function of n degree. The linear combination is responsible for spline function smoothness. Spline functions of integer knots can be interpreted as functions of different resolution in the context of multiscale representation [1]. The base function equation for n = 0 yields $\varphi^0(x/m)$, and is 1 for $x \in [0, m]$ and 0 in other case:

$$\varphi^0(x/m) = \sum_{k=0}^{m-1} \varphi^0(x-k) = \sum_{k \in C} h_m^0(k)\varphi^0(x-k) \tag{2}$$

where $h_m^0(k)$ represents a filter of Z transform $H_m^0(z) = \sum_{k=0}^{m-1} z^{-k}$ so a discrete impulse of length m. The (n+1) convolution of the function reads:

$$H_m^n(z) = \frac{1}{m^n}\left(H_m^0(z)\right)^{n+1} = \frac{1}{m^n}\left(\sum_{m=0}^{m-1} z^{-k}\right)^{n+1} \tag{3}$$

which represents *n+1-th* convolution of discrete pulse and can be implemented as fast algorithms e.g. FIR filters. The coefficients of the filter yields and resemble the Pascal triangle.

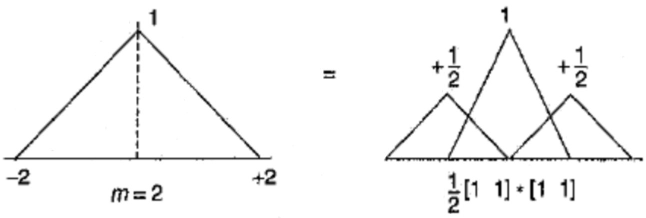

Fig.1. Level of detail process for spline function of degree 1.

This situation is shown in the Fig. 1 for spline function of order 1, as so called spline function pyramid. Eventually, spline function representation for *n-th* order is presented as:

$$\varphi^n(x/m) = \sum_{k \in Z} h_m^n(k)\varphi^n(x-k) \tag{4}$$

and can be interpreted as the spline function hierarchy.

2. SPLINE FUNCTION EDGE MODIFICATION

The hierarchical spline functions of order 1 and 3 are presented in the Fig. 2a and Fig. 2b. In the figures the way of multiresolution approximation construction process is presented. The new set of control points is calculated from control points from the upper scale. The scale change can be introduced as knot removal and a control point reduction. In the first, but important case, base functions are triangles (see Fig. 2a). The figure presents the main idea behind the decimation process as well. The overall process is called a generalization.

a) b)

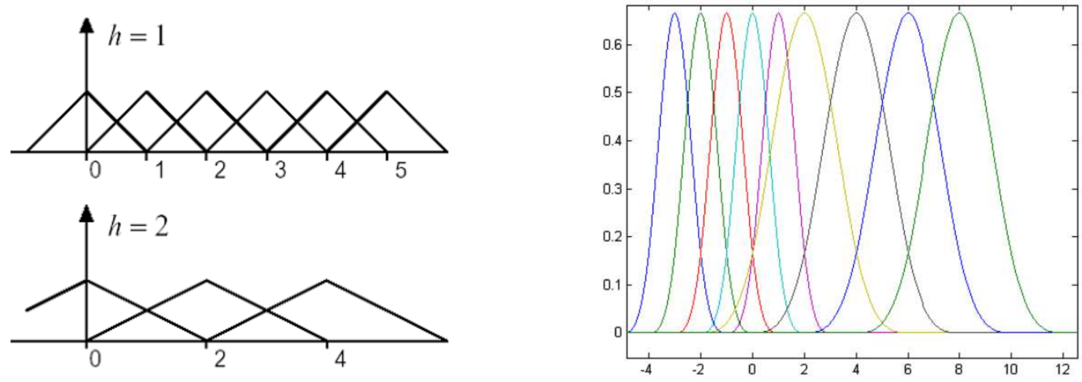

Fig.2a. Knot resolution change in the process of generalization, 2b. Base function of order 3 of mixed resolution.

a) b)

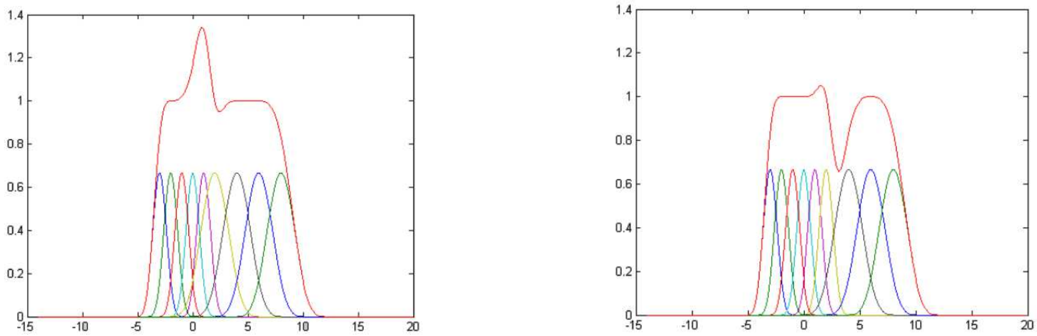

Fig.3a. The oscillation on the edge of different resolution before knot removal, 3b. The oscillation on the edge of different resolution after knot removal.

The control points decimation process means the reducing of the approximated data and the scale changing (see Fig.2a) [3]. The spline function equation (1) yields then:

$$s_h(x) = \sum_{k \in Z^p} c_h(k) \varphi(x/h - k) \qquad (5)$$

where *h* represents scale and φ represents scale function.

The Fig.3. presents a problem which emerge while different domains of different resolution are merged, i.e. oscillation on the stick edges. The Fig.3a presents the situation after knot insertion while Fig.3b, presents the situation after the knot removal. The process of the oscillation compensation consists in de Boor knot insertion. The heuristic algorithm consists of a few steps:

1. Coarse approximation of low resolution data from with spline functions
2. Knot insertion for areas of high resolution
3. Control points modification which corresponds to the high resolution area

Repeated knot insertion for areas of high resolution if the area is not covered by the new set of knots, other situation ends the algorithm.

3. HIERARCHICAL SPLINE MULTIRESOUTION ANALYSIS

The hierarchical spline approach to multiresolution representation can be analyzed using the sampling Nyquist theorem, introduced in the chapter 2 of the paper. The chapter underlines an important aspect of the spline functions analysis, namely their FIR filter implementation possibility. However, in the context of the paper and the 3D data representation, more convenient and general, a geometric approach seems more appropriate.

Let V^k represents control points space of resolution k. There is a transformation which transform one space into another:

$$R^{[k+1]} = SV^{[k]} \tag{6}$$

where S depicts the transformation. One could imagine such representation of different kind of 3D data which are represented in some places by control points of high density as shown in Fig.4. Fig.4 and Fig.5 present low and high control points representation as a regular grid in different colors. If $\vec{O}$ would represent a control points translation in newer space of higher resolution, the translation in the space reads:

$$V^{[k+1]} = R^{[k+1]} \oplus \vec{O}^{[k+1]} \tag{7}$$

where $\oplus$ operator can be interpreted as the transposition $\vec{O}$. The result of the operation was show in the Fig. 5 as high density yellow grid overlaid the base low resolution control points grid.

The hierarchical spline representation may be applied to different kind of data e.g. from bathymetric data of electronic chart (low resolution) to bathymetric data from multibeam sonar (high density resolution) etc.

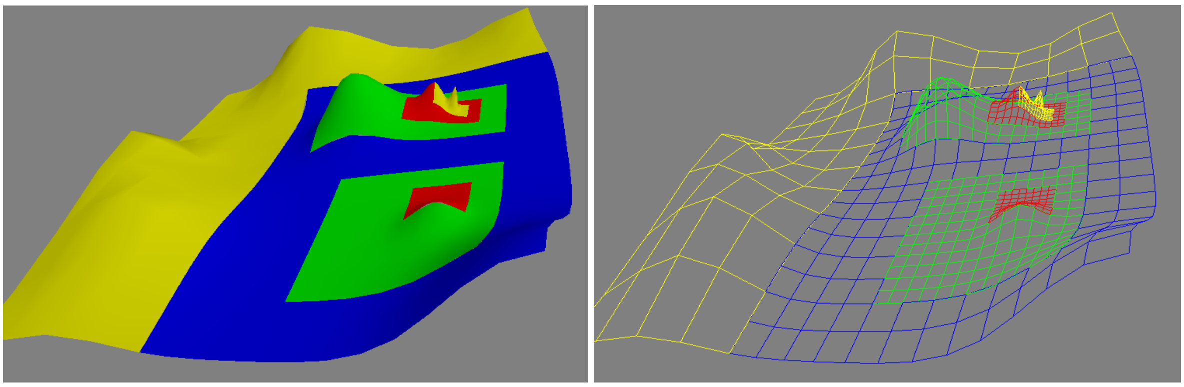

Fig.4. Multiresoution 3D data representation using hierarchical spline function techniques.

The crucial question, which arises at this point refers to the approximation error, as the error could be used by the automatic hierarchical spline grid generation process. The error could be calculated through following mean square error:

$$ER = \sqrt{\frac{\sum_{i=0}^{n}(f(x_i) - s(x_i))^2}{\text{n - m}}} \qquad (8)$$

where $f(x_i)$ represents bathymetric measurement in the point x and $s(x_i)$ is the spline

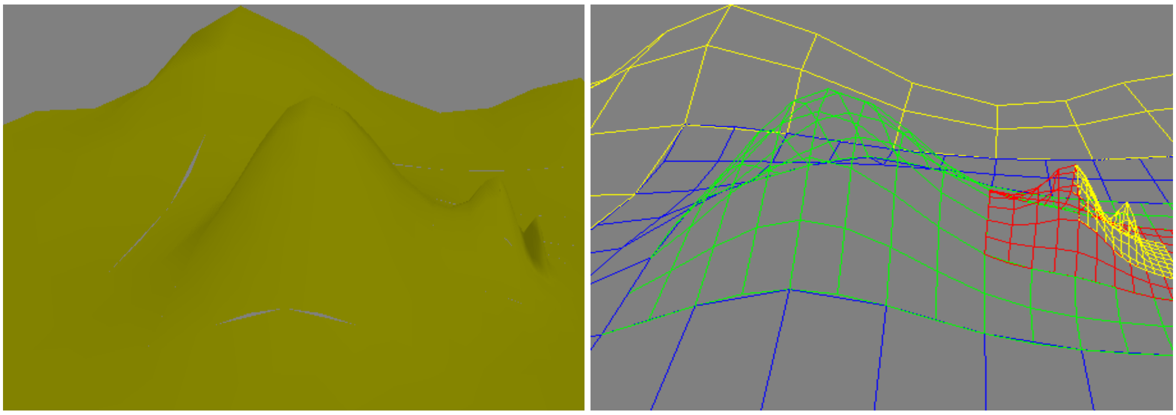

Fig.5. High and low horizontal resolution MBS records approximation using hierarchical spline function.

approximation in the point x. The approximation error depends on the sampling frequency (so the Nyquist theorem) and as a consequence this is a reason the hierarchical splines are so useful. The hierarchical splines can approximate the data taking into account their local density, therefore the local spline approximation error can be very useful in the context of the vertical spline konts resolution estimation.

4. CONCLUSIONS

The problem of the efficient 3D spatial data representation is still opened. There are two main reasons for that: redundancy and the excessive amount of the data. Multibeam sonar data are typical in that context, as multibeam sonar records are an example of a high resolution quasi-raster spatial data. Interpolating and approximating and eventually displaying scattered 3D raster data of high volume leads to some difficulties related to computer processing power. The proposed approach consists of two stages: firstly, all acquired high resolution multibeam sonar raw bathymetric data are interpolated with high density uniform spline interpolation and then knots and control points of interpolated network are saved for defined resolution level and then combined with low resolution data sets [2]. Some redundancy and ambiguity of the measurements is not a drawback in the context of the spline approximation, but it can be treated rather as an advantage and, in fact are indispensable [4]. This flexible approach allows for the spline hierarchical technique usage only in the areas where it is required, namely areas of high horizontal and vertical resolution. At this stage of the investigation, author have implemented algorithms for hierarchical spline representation of 3D spatial data of different resolution. The next stage will include the process automation as applied to the high volume data. This stage will use local approximation error, as a first step of the hierarchical spline local knot resolution determination.

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