A NOVEL METHOD OF TIME-FREQUENCY ANALYSIS: AN ESSENTIAL SPECTROGRAM

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A novel precise method of time-frequency analysis is presented. In the algorithm, a new energy distribution is estimated by simultaneously discard or displacement of the classical spectrogram energy. A channelized instantaneous frequency and a local group delay are used in order to replacement in the same manner as formulated by Kodera et al. [1, 2]. Additionally, new representations: a channelized instantaneous bandwidth and a local group duration are used in order to remove some part of irrelevant energy. A newly obtained energy distribution called ***essential spectrogram*** *is highly concentrated and signal mono-components are precisely localized in the time-frequency domain.*

INTRODUCTION

The Kodera's *et al.* approach of signal analysis is known under many names including: the modified moving window method [1, 2], the cross-spectral method [3], the reassignment method [4, 5], relocation, displacement method, etc. These variants of the method are distinguished mainly by usage of different estimators of the channelized instantaneous frequency (CIF) and the local group delay (LGD) but the main concept of an estimation is the same – energy concentration using CIF and LGD .

Both CIF and LGD are components of the gradient of the STFT complex phase. Other components are the signed channelized instantaneous bandwidth and a signed local group duration. All mentioned components of the gradient of the STFT complex phase are used in the presented approach. Firstly, the short-time Fourier transform (STFT) is derived in the following manner:

$$U(t,\omega) = A(t,\omega)\exp\big(j\phi(t,\omega)\big) = \int_{-\infty}^{\infty} u(\tau + t)h^*(-\tau)\exp(-j\omega\tau)d\tau \tag{1}$$

where complex conjugation is denoted by an asterisk,

$$A(t,\omega) = \big|U(t,\omega)\big| \quad \text{and} \quad \phi(t,\omega) = \arg\{U(t,\omega)\}\,, \quad A(t,\omega),\ \phi(t,\omega) \in \mathbb{R} \tag{2}$$

The complex waveform denoted by $u(t)$ should have non-zero values and has to be differentiable in every instant, $U(t,\omega)$ means resultant STFT and $h(t)$ represents an analyzing window function. $A(t,\omega)$ and $\phi(t,\omega)$ denote accordingly amplitude and phase instantaneous spectra. Subsequently for each locus (t,ω) of STFT is estimated corrected localization in the time-frequency plain by CIF and LGD, that are expressed respectively:

$$\Omega(t,\omega) = \frac{\partial}{\partial t}\phi(t,\omega) \tag{3}$$

$$\Theta(t,\omega) = -\frac{\partial}{\partial \omega}\phi(t,\omega) \tag{4}$$

and using for obtained new localizations as follows:

$$(t,\omega) \rightarrow \big(t + \Theta(t,\omega)/(2\pi), \Omega(t,\omega)\big) \tag{5}$$

where t and ω mean accordingly time and angular frequency. CIF is denoted by $\Omega(t,\omega)$ and LGD is referred to as $\Theta(t,\omega)$. The new distribution of energy is called concentrated spectrogram [8]. In the Fig. 1 classical and concentrated spectrograms of a constant-envelope LFM chirp are presented. A realization of weak white Gaussian noise is added to the testing signal, SNR is equal approx. 30 dB.

In classical and concentrated spectrograms two mono-components interfere significantly, if they are located close to each other on the time-frequency plain. The phenomenon is described by the uncertainty principle. Thus the interferences of components occurs locally. They are dependent on the time-frequency range of the analyzing window that is represented by the unambiguity function [6, 7]. In the Fig. 2 classical and related concentrated spectrograms of a two-component chirp signal are presented. Weak white Gaussian noise is added to the testing signal, SNR is equal approx. 30 dB.

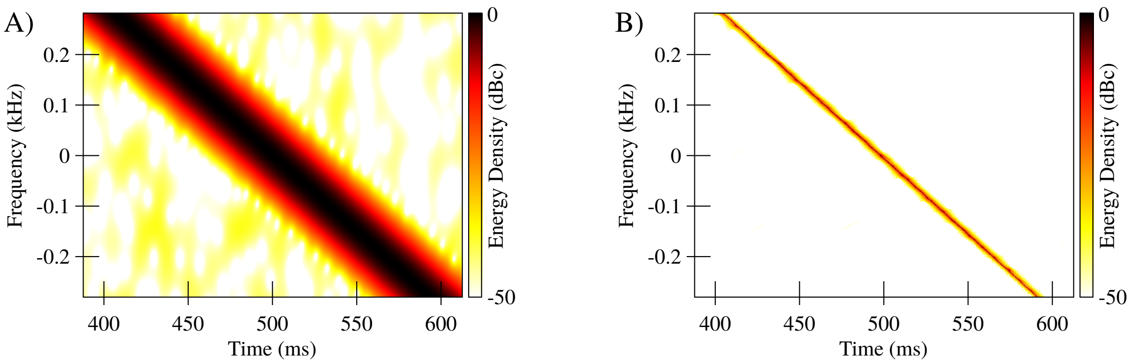

Fig. 1. Energy distributions of noised LFM chirp signal in the time-frequency domain: A) classical spectrogram B) concentrated spectrogram. SNR is equal approx. 30 dB.

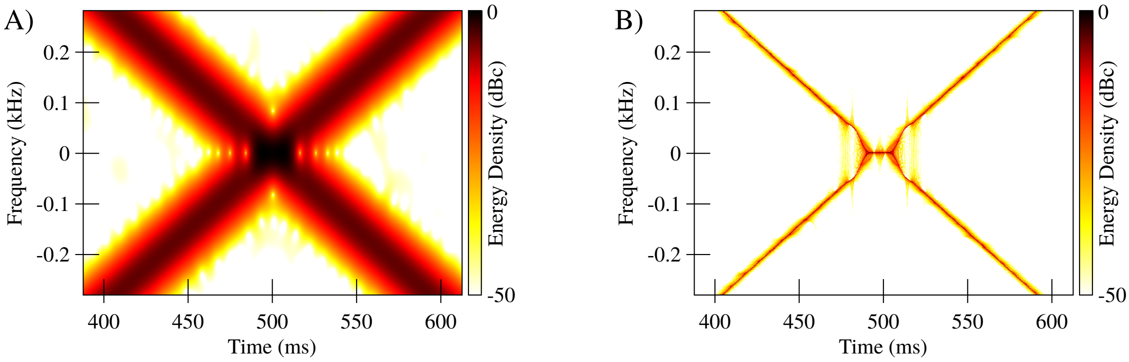

Fig. 2. Energy distributions of two crossed LFM chirps signal in the time-frequency domain: A) classical spectrogram B) concentrated spectrogram. SNR is equal approx. 30 dB.

In the next sections of the paper, a number of degrees of freedom distribution estimate as a product of the channelized instantaneous bandwidth (CIBW) and a local group duration (LGDR) is introduced. This representation is used in order to select the areas where energy is originated mainly from a single signal mono-component. The energy is extracted from the spectrogram and located according to the Kodera's *et al.* approach.

1. DEGREES OF FREEDOM IN SIGNAL THEORY

A number of degrees of freedom for any signal denoted here by χ_e can be calculated as a product of its effective bandwidth and its effective duration as follows [10]:

$$\chi_\mathrm{e} = B_\mathrm{e} T_\mathrm{e} \tag{6}$$

where B_e and T_e represent respectively the effective bandwidth:

$$B_\mathrm{e}^2 = \int_{-\infty}^{\infty} (\omega-\omega_o)^2 E(\omega)\mathrm{d}\omega \Big/ \int_{-\infty}^{\infty} E(\omega)\mathrm{d}\omega\,, \quad \omega_o = \int_{-\infty}^{\infty} \omega E(\omega)\mathrm{d}\omega \Big/ \int_{-\infty}^{\infty} E(\omega)\mathrm{d}\omega \tag{7}$$

and the effective duration of the signal:

$$T_\mathrm{e}^2 = \int_{-\infty}^{\infty} (t-t_o)^2 E(t)\mathrm{d}t \Big/ \int_{-\infty}^{\infty} E(t)\mathrm{d}t\,, \quad t_o = \int_{-\infty}^{\infty} tE(t)\mathrm{d}t \Big/ \int_{-\infty}^{\infty} E(t)\mathrm{d}t \tag{8}$$

Both B_e and T_e are always positive and χ_e limited: $0 < B_\mathrm{e}$, $0 < T_\mathrm{e}$ and $\chi_\mathrm{e} < \infty$. $E(t)$ denotes a distribution of signal energy in the time domain and $E(\omega)$ represents a distribution of spectrum energy in the frequency domain. The number of degrees of freedom (6) is wider known as the time-bandwidth product and is used in order to evaluate of analyzing windows.

2. LOCAL NUMBER OF DEGREES OF FREEDOM

The local bandwidth can be assigned in every instant and in every output channel from the short-time Fourier transformer, similarly as the channelized instantaneous frequency [11]. Then for both continues time and frequency it is called the channelized instantaneous bandwidth and can be obtain as follows:

$$B(t,\omega) = \frac{1}{2\pi}\left|\frac{\partial}{\partial t}\Lambda(t,\omega)\right| \tag{9}$$

where $\Lambda(t,\omega) = \ln\{A(t,\omega)\}$ and $\ln\{\}$ is the complex natural logarithm operator. Dually, a local group duration can be defined in the time-frequency domain:

$$T(t,\omega) = \frac{1}{2\pi}\left|\frac{\partial}{\partial \omega}\Lambda(t,\omega)\right| \tag{10}$$

While the channelized instantaneous bandwidth and the local group duration express a local stretching of the signal respectively in frequency and in time. A number of degrees of freedom distribution can be estimated by the following formula:

$$\chi(t,\omega) = B(t,\omega)T(t,\omega) \tag{11}$$

where $\chi(t,\omega)$ is the local number of degrees of freedom (LNDF) distributed in the join time-frequency domain. To distinguishing from the global number of degrees of freedom (6) the local value is denoted by $\chi(t,\omega)$ without any subscript. In the Fig. 3. LNDF distributions of the test chirp signals are presented.

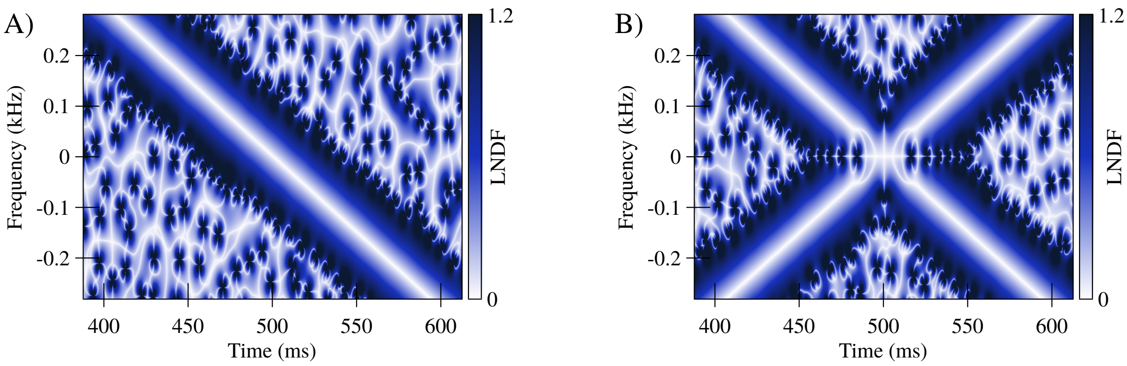

Fig. 3. Local number of degrees of freedom obtain for: A) LFM chirp B) cross-chirp. SNR is equal approx. 30 dB.

3. ENERGY EXTRACTION FROM A SPECTROGRAM

Both classical and concentrated spectrograms are some estimates of an energy distribution in the time-frequency domain. The main purpose of the proposed method is a classification of the energy in order to extract unambiguous not smeared part. It is assumed that the not smeared energy expresses a true localization of signal components in the time-frequency plain. The smeared energy can be detected using LNDF. If local number of degrees of freedom is large, a time-frequency representation (TFR) of a signal is chaotic and it is rapidly changing. The estimated in a point LNDF concerns area that can be indicated by the unambiguity function of an analyzing window near the point. Contrary, if a local number of degrees of freedom achieves small value, then TFR is locally orderly and slowly variable.

In order to an energy separation a threshold of LNDF can be arbitrarily assumed. Let the threshold be denoted as α_χ. Then if LNDF is greater than α_χ, an energy of a classical spectrogram is removed from this point of time-frequency plain. Otherwise an energy is preserved. The remaining energy is subsequently redistributed according to the Kodera's *et al.* approach. The resultant energy distribution is called **the essential spectrogram**. Essential spectrograms of the test chirp signals are illustrated by the Fig. 4. What causes a great impression is a high concentration of the energy for both mono- and two-component signals [9]. The Heisenberg-Gabor uncertainty is responsible for characteristic deformations in a center of the presented spectrogram in the Fig. 4.B and 5.B.

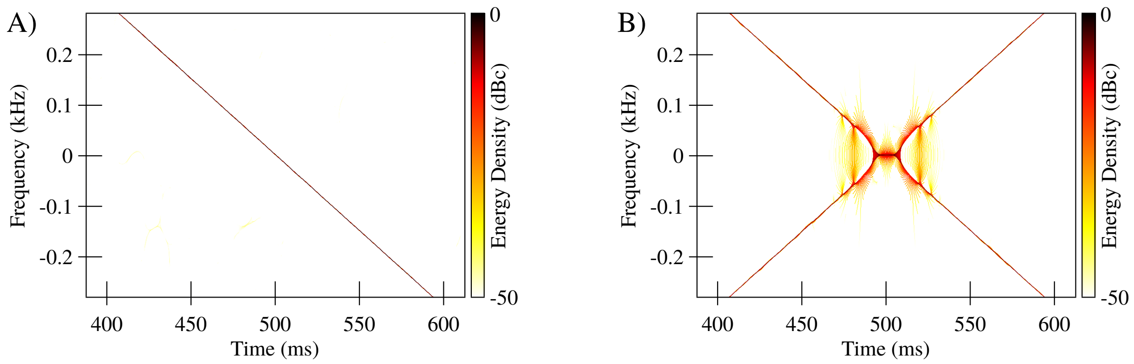

Fig. 4. Essential spectrograms of: A) LFM chirp B) cross-chirp. SNR is equal approx. 30 dB and the threshold α_χ is assumed as 0.8.

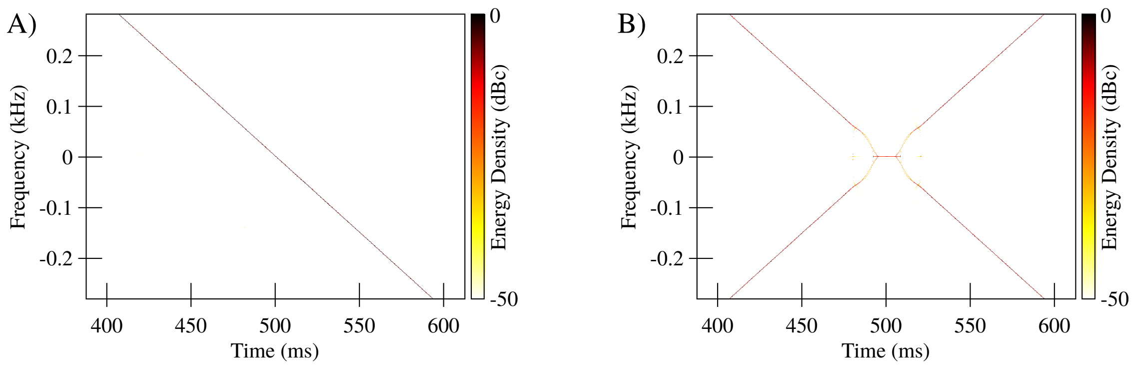

Fig. 5. Essential spectrograms of: A) LFM chirp B) cross-chirp. SNR is equal approx. 30 dB and the threshold α_χ is assumed as 0.002.

4. CONCLUSION

In the paper, the novel method of an energy distribution estimation is presented. The resultant energy distribution is called **the essential spectrogram**. A new aspect of the method is used the local number of degrees of freedom (LNDF) distributed in the join time-frequency domain in order to separate energy into two parts. LNDF distribution is obtain as a product of the channelized instantaneous bandwidth and a local group duration. The part of energy, where LNDF values are small, is referred to as the essential spectrogram. The localization of the energy is calculated according to the Kodera *et al.* approach. Whereas, the second part is treated as an irrelevant effect of the short-time Fourier transformation and it is strongly dependent on the Heisenberg-Gabor principle. The spectrograms in the Fig. 4. prove that proposed energy distributions are high concentrated and accurate in the time-frequency domain more than classical and concentrated spectrograms.

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