MODELING OF THE CONSTANT SOUND SPEED SURFACE IN WATER USING BICUBIC HERMITE'S PIECES

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In the paper the interpolation method of two variables using bicubic Hermite's pieces has been shown. Basics preceded theory of Coons' pieces and bicubic one, which are the introduction to interpolation method of bicubic Hermite's pieces. For determination the surface using this method hypothetical data have been used. As an example of the small area with differences of the vertical distribution of the sound speed in water estuary of the Vistula river has been assumed.

INTRODUCTION

The problem of modeling a surface has many applications. In the hydrography area is used for modeling the sea bottom, for determination contour lines and today for spatial presentation in ECDIS (Electronic Chart Display and Information System). TIN (Triangulated Irregular Network) and GRID with many interpolation methods have been used. Today commonly is used mathematical modeling using methods: B-spline [2,3,8,10,11,13], NURBS (Non Uniform Rational B-Splines) [9,11,12,14] and Bézier [1,4,5,6,7], Bernstein's functions and other. One of them is the Coons' pieces method [1], which theoretical basics and application have been presented.

1. COONS' PIECES

Coons' piece is described by following relationship [1]:

$$p_1(u,v) = c_{00}(u)H_{00}(v) + c_{10}(u)H_{10}(u)H_{10}(v) + c_{0r}(u)H_{0r}(v) + c_{1r}(u)H_{1r}(v) \tag{1}$$

where H_{mj} are Hermite's basis polynomials of $2r+1$ degree given for knots 0 and 1 and

$$c_{00}(u),...,c_{0r}(u) \quad \text{oraz} \quad c_{10}(u),...,c_{1r}(u) \tag{2}$$

are curves, which creates the piece. Moreover they fulfill interpolation conditions [1]:

$$\begin{array}{ll} p_1(u,v)\big|_{v=0} = c_{00}(u) & p_1(u,v)\big|_{v=1} = c_{10}(u) \\ \quad\cdots & \quad\cdots \\ \dfrac{\partial^r}{\partial v^r} p_1(u,v)\Big|_{v=0} = c_{0r}(u) & \dfrac{\partial^r}{\partial v^r} p_1(u,v)\Big|_{v=1} = c_{1r}(u) \end{array} \tag{3}$$

Bicubic Coons' piece arising in case of r=1 and is given by describing four border curves and four curves describing partial derivatives in transverse direction to the edge. The name "bicubic" treats to the degree of Hermite's basis polynomials. To conditions of positioning conformity, which must be fulfilled by edge curves, arrive conditions of derivatives conformity, among other things:

$$\begin{array}{l} c'_{00}(0) = d_{01}(0) = p_u(0,0) \\ d'_{00}(0) = c_{01}(0) = p_v(0,0) \end{array} \tag{4}$$

Piece of Coons' surfaces are created using Hermite's bases of 5th and higher degrees are used relatively seldom, because there is necessary to give big number of curves and to assure fulfilling by them conformity conditions.

2. BICUBIC PIECES IN HERMITE'S REPRESENTATION

If every curves used for construction bicubic Coons' piece are polynomial of non higher than 3 degree, so turns out that $p = p_1 = p_2 = p_3$.

It follows that matrixes $C(u)$ and $D(v)$ created by curves are described by equations

$$C(u) = H(u)P \tag{5}$$

$$D(v) = H(v)P^T \tag{6}$$

with matrix P, which coefficients are end points and vectors curves' derivatives (and finally the piece) [1]:

$$P = \begin{bmatrix} p(0,0) & p(0,1) & p_v(0,0) & p_v(0,1) \\ p(1,0) & p(1,1) & p_v(1,0) & p_v(1,1) \\ p_u(0,0) & p_u(0,1) & p_{uv}(0,0) & p_{uv}(0,1) \\ p_u(1,0) & p_u(1,1) & p_{uv}(1,0) & p_{uv}(1,1) \end{bmatrix} \tag{7}$$

For unambiguous description of the piece there is enough to give:

- four corner points of the piece,
- eight vectors of partial derivatives (two vectors in each corner),
- four vectors of mixed derivatives (one vectors in each corner).

All in all there are 16 (vector) parameters. Because the piece is unambiguous described by Hermite's interpolation conditions, so this characteristic case of Coons' surface is often described as bicubic piece in Hermite's representation.

If Hermite's representation of bicubic piece is known, there is possible to point at its Bézier's representation. If by Q we mark the matrix, which elements are control points of Bézier's piece of (3,3) degree, that

$$p(u,v) = H(u)PH(v)^T = B(u)QB(v)^T \tag{8}$$

where matrix $B(u) = [B_0^3(u),\ B_1^3(u),\ B_2^3(u),\ B_3^3(u),]$ represents the base pf Bernstein's polynomials. Because Hermite's base and Bernstein's one are connected by equation

$$H(u) = B(u)A \tag{9}$$

in which

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

so

$$Q = APA^T \tag{11}$$

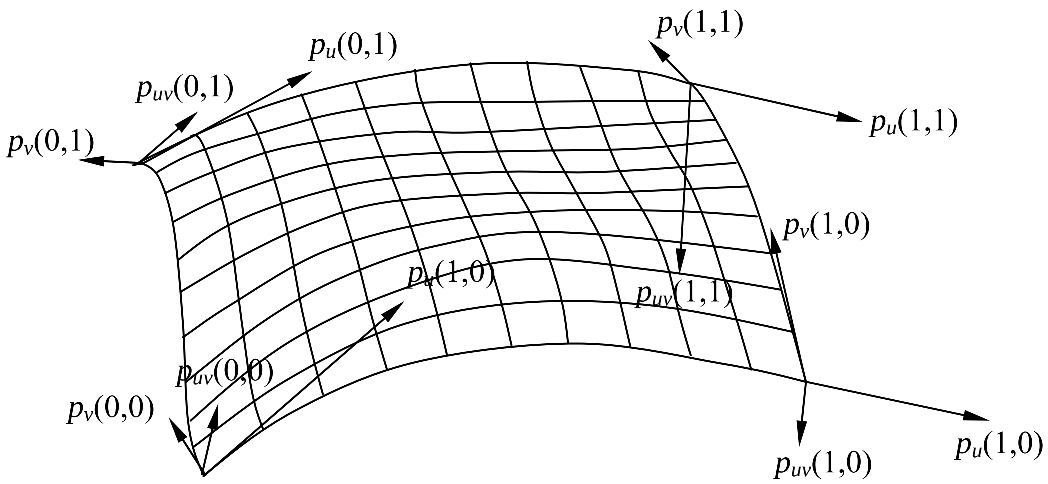

Fig.1. Polynomial piece of (3,3) degree in Hermite's representation and defined it parameters. Vectors of derivatives are scaled by value 1/3, mixed derivatives by 1/9.

3. RESULTS

For modeling the surface of sound speed in water the hypothetical data with high differences of the vertical distribution on small area, such as estuary of Vistuls river, have been shown. In this area salt and cold water in the Gulf of Gdansk is supplied by fresh and warm water from the river. The phenomenon of the vertical distribution of the sound speed's changes can be observed in other areas, but on longer distances. In areas with spatial unchangeable vertical distribution we observe only time changes that is fluctuation of the surface of the sound speed in water.

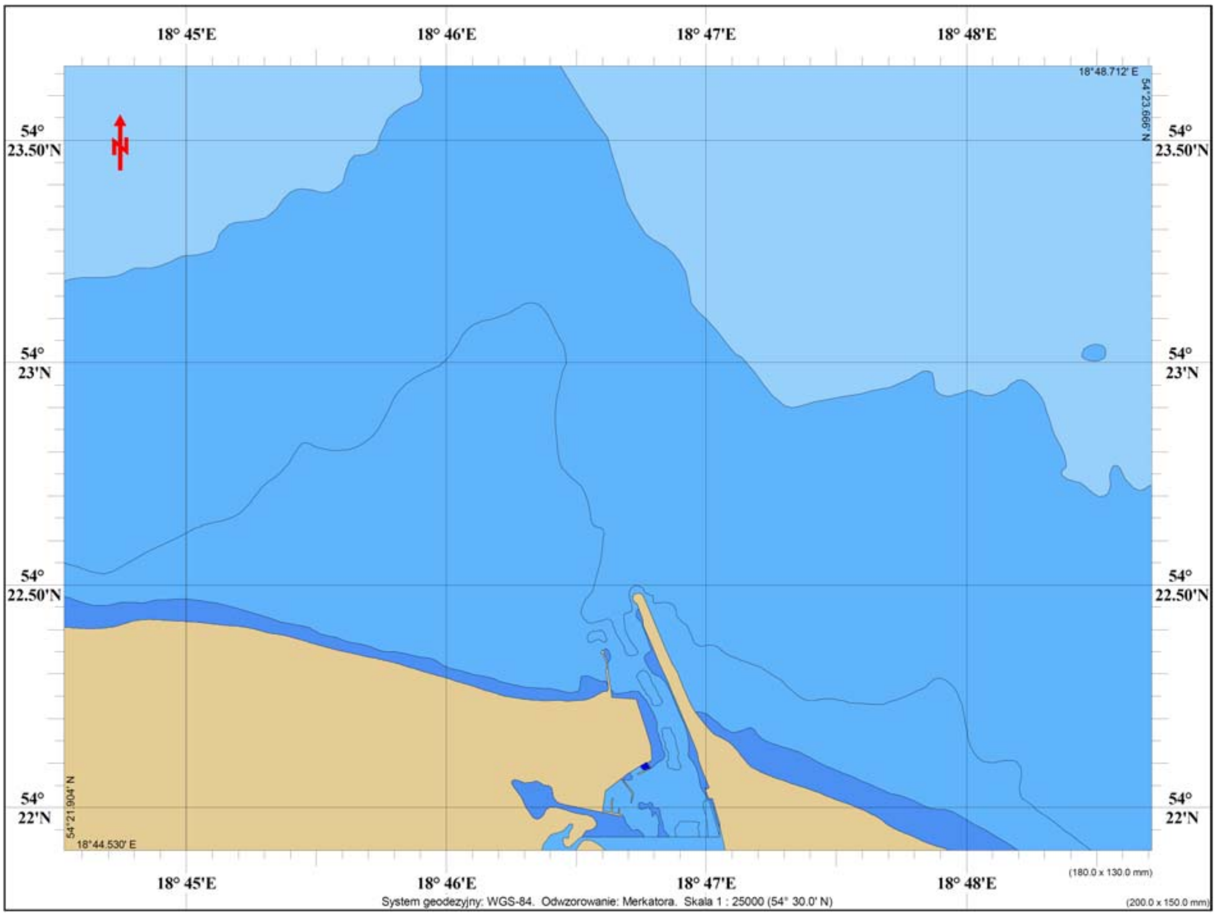

Fig.2. Hypothetical area of sound speed measurements.

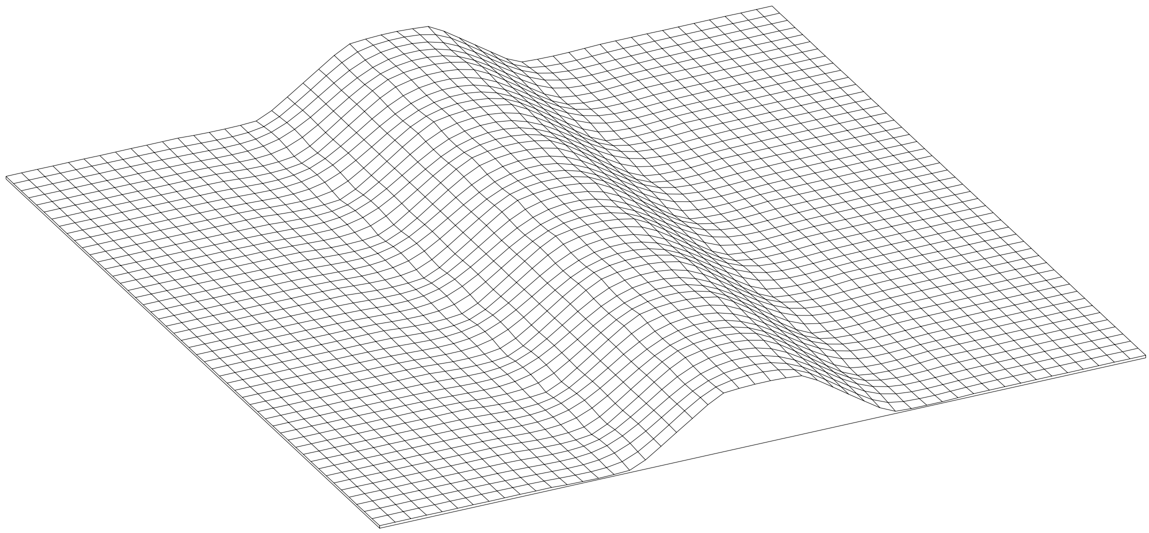

Fig.3. Surface of the sound speed modeled using bicubic Hermite's pieces.

4. CONCLUSIONS

Pieces and bicubic one in Hermite's representation are another interpolation methods for modeling the surface. One of applications in hydrography and hydroacoustics can be determination the surface of the sound speed in water on the basis of simultaneously measurements of their vertical distributions.

Another step in application of the spatial distribution of the sound speed in water can be research of time changes of fluctuating surfaces.

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