APPROXIMATION OF VERTICAL DISTRIBUTION OF THE SOUND SPEED IN WATER USING BASIS HERMITE’S POLYNOMIAL

ARTUR MAKAR

Polish Naval Academy
69, Śmidowicza St., 81-103 Gdynia, Poland
A.Makar@amw.gdynia.pl

In the paper theoretical basics of interpolation method of one variable using basis Hermite’s polynomials have been presented. There have been described local Hermite’s bases and polynomial interpolation relationship. The metgod has been used for description the vertical distribution of the sound speed in water in Five Whistles Corner area in the Motlawa estuary in Gdansk Nowy Port.

INTRODUCTION

Representation of curves and surfaces polynomial and piece polynomial using bases of Bernstein polynomials or B-splines is suitable during shaping curves and surfaces without taking note on interpolation conditions. In practice often occurs the situation, when there is necessary to create the smooth curve crossing specific sequence of points. Interpolation B-spline curve can be obtained by solution of the set of linear equations, which gives control points of the curve. Another representation of the curve can be Hermite’s representation, used for modeling of the vertical distribution of the sound speed in water.

1. LOCAL HERMITE'S BASES

Let us take into consideration established sequence of numbers $u_0,...,u_N$, which we assume as interpolation knots. For each knot we define the value of the function or point of the curve, which should be constructed. Simultaneously they will be knots of function or splined curve, that is connecting points of polynomials or polynomial arc of the curve.

For each segment $[u_i,u_{i+1}]$ we define polynomial base, which will be used for local description of the curve on this segment. Starting point is the problem of Hermite’s interpolation with two $(r+1)$-times interpolation knots u_i i u_{i+1}. We want to obtain the base of polynomial space of $2r+1$ degree at the most

$$\left\{H_{i,00}, H_{i,10}, \ldots, H_{i,0r}, H_{i,1r}\right\} \tag{1}$$

such as the polynomial $p(t)$, which is the solution of interpolation problem with given values $p(u_i)$, $p(u_{i+1})$, $p'(u_i)$, $p'(u_{i+1})$,..., $p^{(r)}(u_i)$, $p^{(r)}(u_{i+1})$, can be written in the form:

$$p(t) = p(u_i)H_{i,00}(t) + p(u_{i+1})H_{i,10}(t) + p^{(r)}(u_i)H_{i,0r}(t) + p^{(r)}(u_{i+1})H_{i,1r}(t) \tag{2}$$

This polynomial base we will call Hermite's local base. Description "Hermite's local base" and Hermite's representation" lean on the relationship the base with the Hermite's interpolation problem. They have nothing to do with Hermite's orthogonal polynomials.

There is easy to notice, that for each r the base given in selected method has following property:

– One distribution: $H_{i,00}(t) + H_{i,10}(t) = 1$.
– Symmetry: $H_{i,0j}(U_i+t) = (-1)^j H_{i,1j}(u_{i+1} - t)$.
– Relationship between different bases: Let $\{H_{00}, H_{10}, \ldots, H_{0r}, H_{1r})$ denotes the local base determined by knots 0 and 1. Then for any $u_i < u_{i+1}$, $m \in \{0, 1\}$, $j = 0, \ldots, r$, there is equality $H_{i,mj}(t) = h_i^j H_{mj}(u)$; $h_i = u_{i+1} - u_i$, $u = (t - u_i)/h_i$.

Finding Hermite's local base can be realized as finding the base for knots 0 and 1. So as to find the polynomial H_{00}, there is enough to solve the interpolation problem assuming $H_{00}(0) = 1$, $H_{00}(1) = 0$ and zero values of derivatives of 1,...,r degree in both knots; finding the base is realized as solving $2r+2$ interpolation problems.

For r= 0 there is the problem of Lagrange's interpolation. Basis polynomials have following form:

$$H_{00}(u) = 1 - u \tag{3}$$

$$H_{10}(u) = u \tag{4}$$

For r=1 the local Hermite's base for knots 0 and 1 consists of following polynomials:

$$H_{00}(u) = 2u^3 - 3u^2 + 1 \tag{5}$$

$$H_{01}(u) = u^3 - 2u^2 + u \tag{6}$$

$$H_{10}(u) = -2u^3 + 3u^2 \tag{7}$$

$$H_{11}(u) = u^3 - u^2 \tag{8}$$

Local Hermite's base for r=2 consists of following polynomials:

$$H_{00}(u) = (6u^2 + 3u + 1)(1 - u)^3 \tag{9}$$

$$H_{01}(u) = (3u^2 + u)(1 - u)^3 \tag{10}$$

$$H_{02}(u) = \frac{1}{2}u^2(1 - u)^3 \tag{11}$$

$$H_{10}(u) = u^3(6u^2 - 15u + 10) \tag{12}$$

$$H_{11}(u) = u^3(1 - u)(3u - 4) \tag{13}$$

$$H_{12}(u)=\frac{1}{2}u^3(1-u)^2 \tag{14}$$

For construction the curves and surfaces usually bases of 3rd degree are used (r=1).

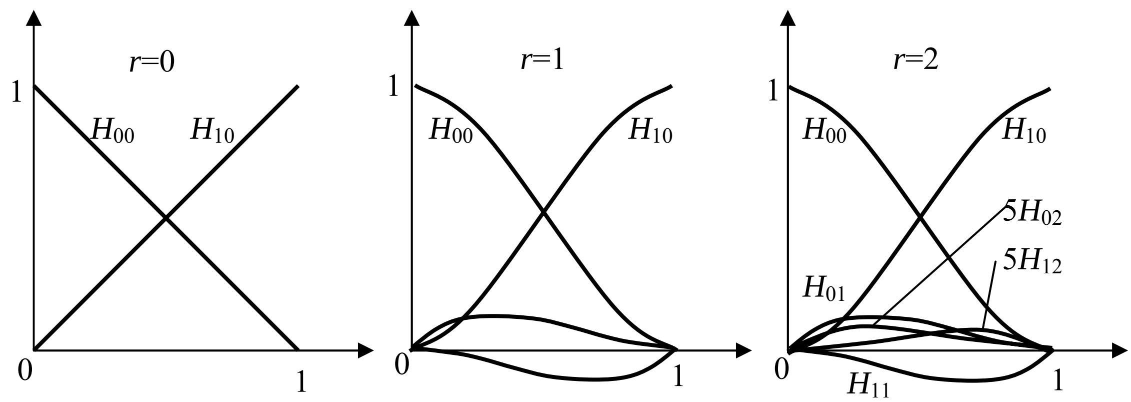

Fig.1. Graphs of Hermite's base polynomials of 1st, 3rd and 5th degree.

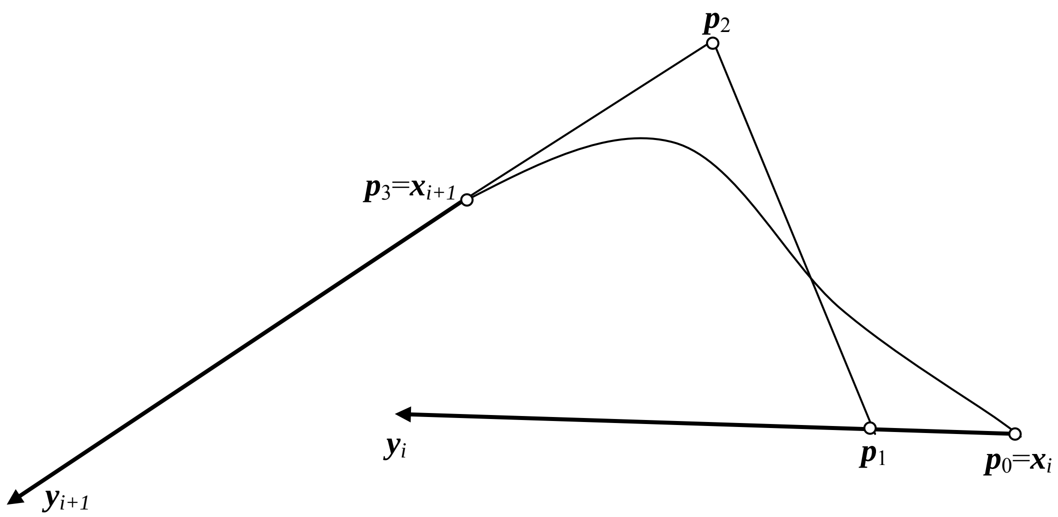

Fig.2. Representations of Bezier's curve and Hermite's one of 3rd degree.

2. RESULTS

For determination the model of the vertical distribution of the sound speed in water have been used measurements realized in Five Whistles Corner area in the Motlawa estuary in Gdansk Nowy Port. Measurements have been realized during sounding Wisloujscie Fortress pound – the area placed close to Five Whistles Corner. Motlawa river, from the crane to the crane is the area with high intensity of traffic, mainly ferries, large ships and pleasure-boats.

The area of measurements Has been show in Fig. 3.

Fig.3. Sound speed measurements area.

Representation of Hermite's curve of 3rd degree requires points x_i and x_{i+1} of derivative's vectors y_i and y_{i+1} for the valure of parameter t. Changing symbols of variables in common theory of modeling for variables used for description the vertical distribution of the sound speed in water, the curve in the base of 3rd degree is described by following equation:

$$c(h) = H_{i,00}(h)x_i + H_{i,01}(h)y_i + H_{i,10}(h)x_{i+1} + H_{i,11}(t)y_{i+1} \tag{15}$$

For the arc of the polynomial curve of 3rd degree, in Hermite's representation, Bezier's control points can be easily shown.

In Fig. 4 real measurements of the vertical distribution of the sound speed in water and their Hermite's representation for r=2 have been shown.

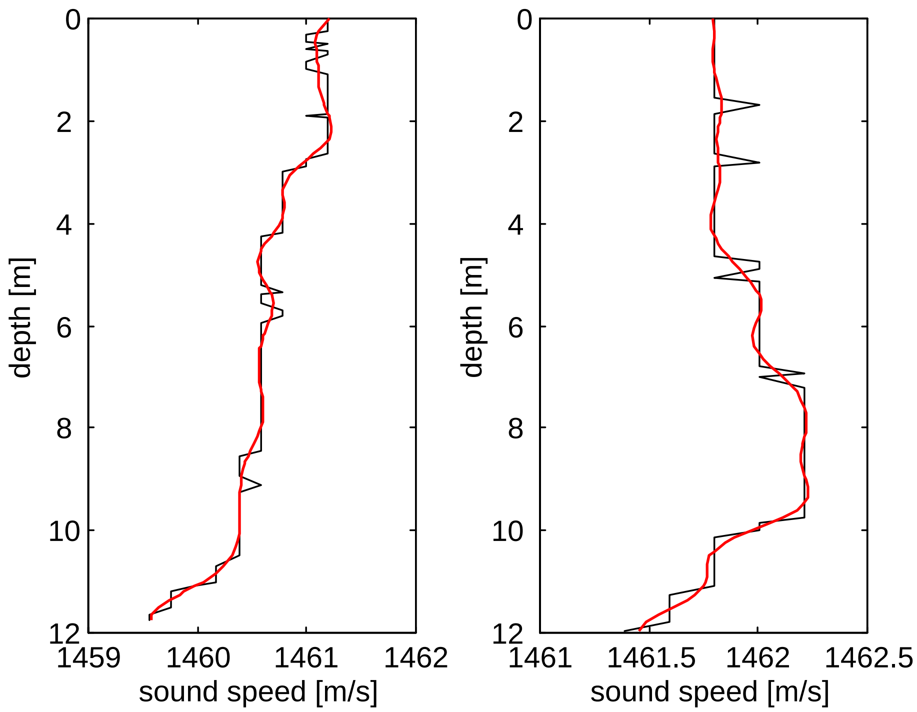

Fig.4. Real measurements of the vertical distribution of the sound speed in water and their Hermite's representation for r=2.

3. CONCLUSIONS

Presented method of modeling of the curve is another one, one of many interpolation methods. Interpolation of the vertical distribution of the sound speed in water is the essential problem in hydrography during reduction of measurements in post processing on the basis of its time and spatial changes. Use of time measurements (in different positions) and time one (in different time) can be used for creation the model of spatial and time distribution of the sound speed in water for its local determination.

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