

# ON THE CHARACTERISTICS OF SOUND GENERATED BY BUBBLES INJECTED UNDERWATER

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*The transient oscillation of an air bubble injected underwater is investigated. This oscillation generates a decaying acoustic signal that can be remotely received and analyzed. The narrowband spectrum of this signal is centered at the bubble oscillation frequency. These results are extended to a plume of bubbles generated by a linear array of nozzles. This models bubble behavior in applications such as passive diver localization, bubble depth measurement, detection of ocean gases, and gas pipe leakage. The results presented show that these bubbles can generate a sufficiently strong acoustic signal that can be utilized for passive remote sensing.*

## INTRODUCTION

Ocean and marine scientists and engineers are interested in sea characteristics and entrained bubbles in an underwater environment for various applications such as methane gas detection and target localization [1]. There are various natural and manmade bubble sources in the ocean. A significant number of bubbles are created on the ocean surface by the breaking action of ocean waves [20]. Raindrops falling on the sea surface are another natural source of bubbles. Cavitation caused by the propeller of a ship or submarine is a source of manmade bubbles. The oscillatory motion of the surface of bubbles entrained in water or other liquids can generate acoustic pressure. The acoustic signal of an air bubble formed at a nozzle was

first investigated by Minnaert [2] and subsequently by Meyer and Tamm [5]. They showed experimentally that an injected bubble behaves like a simple oscillating system with damping. Recently, Pumphrey and Crum [8] measured the oscillation frequency of bubbles near the surface of a water tank. Eller and Pfriem [4,19] examined the damping mechanism of acoustic signals generated by bubbles oscillating in the frequency range 1-50 kHz. They considered viscosity, radiation and thermal damping. The oscillation frequency was related to the ambient pressure and bubble radius as first described by Minnaert [2]. The dynamics of a single bubble are examined in this paper. The results obtained are then extended to a bubble plume or cloud. A method is proposed for bubble depth estimation based on the received acoustic signal. This has many potential underwater applications.

## 1. SINGLE BUBBLE OSCILLATION

The acoustic signal from a single bubble injected into a liquid is related to its surface oscillations. This transient behavior is caused by volume pulsations and shape oscillations. The equation for the radius  $r$  of a bubble was first developed by Rayleigh [21] as

$$r\ddot{r} + \frac{3}{2}\dot{r}^2 = p_i - p_e \quad (1)$$

where  $p_i$  is the pressure inside the bubble and  $p_e$  is the external pressure exerted by the liquid. Strasberg and Fitzpatrick [23] have shown that the instantaneous pressure  $p_s(t)$  at a distance  $d$  from an oscillating bubble is given by

$$\ddot{r}(t') = \frac{1}{\rho} \sqrt[3]{3p_s(t)d} \quad (2)$$

where  $\ddot{r}(t')$  is the second derivative of the instantaneous radius at time  $t' = t - d/c$  and  $\rho$  is the density of the liquid. Lamb [12] used a theoretical method to show that the bubble oscillations are a sum of harmonics. The radius is then given by

$$r(t, \theta, \varphi) = r_0 + \sum_n r_n S_n(\theta, \varphi) e^{2\pi i f_n t} \quad (3)$$

where  $r_0$  is the mean radius of the spherical bubble given by its average radius (over sufficiently large  $t$ ) at a depth  $h$ ,  $S_n$  is the surface harmonic of order  $n$  with azimuth and elevation angles  $\theta$  and  $\varphi$ , respectively,  $r_n$  is the amplitude of oscillation associated with order  $n$ , and  $f_n$  is the corresponding frequency of oscillation. The bubble surface oscillations for modes  $n = 0, 1, 2$  given by (3) are independent of the azimuth angle  $\theta$ , as shown in Fig. 1.

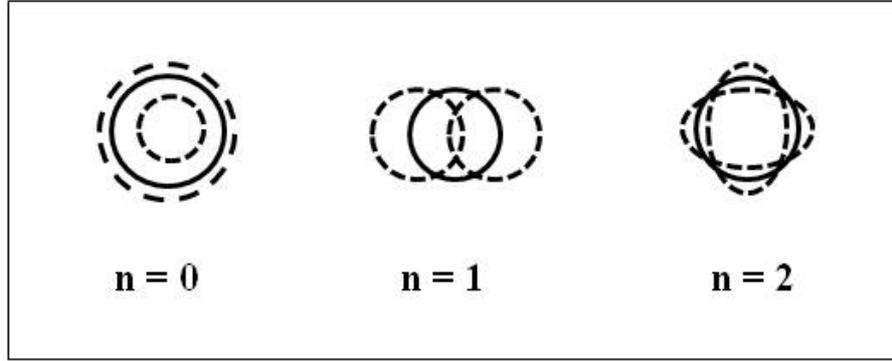


Fig.1. Bubble oscillations for bubble for modes  $n = 1, 2, 3$ .

Order  $n = 0$  corresponds to the volume oscillations with a fixed spherical shape. The natural frequency for this order is called the Minnaert frequency [2, 15] and is given by

$$f_0 = \frac{1}{2\pi r_0} \sqrt{\frac{3\gamma P}{\rho}} \quad (4)$$

where  $P$  is the ambient pressure,  $\gamma$  is the ratio of specific heats of the air inside the bubble to that of the surrounding water and  $\rho$  is the density of the liquid. The  $n = 1$  order (mode 1) corresponds to translational oscillation and  $\rho$  is not associated with a restoring force, so it does not generate a signal. The higher modes ( $n \geq 2$ ) may co-exist simultaneously. Lamb [12] evaluated the higher mode frequencies  $f_n$ , ( $n \geq 2$ ) as

$$f_n = \frac{1}{2\pi r_0} [(n^2 - 1)(n + 2)\sigma / \rho r_0]^{\frac{1}{2}} \quad (5)$$

where  $\sigma$  is the surface tension, which can be neglected for bubbles with radius  $r_0 > 0.1$  mm in pure water. However, this may need to be considered for smaller radii if surfactants are present as they change the bubble surface tension [7]. The lowest surface oscillation frequency corresponds to  $n = 2$  in (5) and can be expressed as

$$f_2 = \frac{1}{2\pi r_0} \sqrt{\frac{12\sigma}{\rho r_0}} \quad (6)$$

Comparing (4) and (6) indicates that volume oscillations ( $n=0$ ) have a much higher frequency than the surface oscillations ( $n \geq 2$ ). A bubble injected into water generates a damped oscillatory pressure signal (a sound pulse) with frequency  $f_0$  given by (4) and duration  $\tau$  determined by the damping. In this paper,  $\tau$  is defined as the time it takes for the pulse envelope to decline to 10% of its maximum (at  $t=0$ ) value.

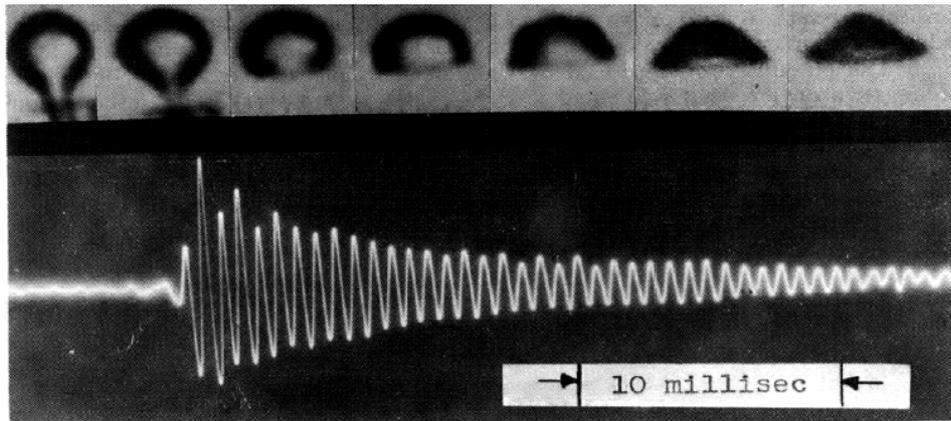


Fig.2. Acoustic pressure decay for a single air bubble formed at a nozzle [23].

Strasberg and Fitzpatrick [23] examined this effect experimentally, and a typical signal is shown in Fig. 2. The top portion of the figure shows the bubble shape over time, and the bottom portion shows the corresponding pressure signal. The oscillatory behavior begins just after the bubble leaves the nozzle and continues for over 20 ms at a frequency of approximately 1.5 kHz. This behavior has been analyzed experimentally by Frizell and Arndt [24] for a single bubble oscillating at a frequency of 2.5 kHz. The bubble behavior is illustrated in Fig. 3. First The bubble begins to form at the nozzle. At time  $t=0.0$  ms, the bubble detaches from the nozzle and begins to oscillate. These oscillations cease at some time  $\tau$ . After  $t>\tau$  the bubble rises in the water column without oscillations at a velocity based on the buoyancy, water drag and the ambient pressure.

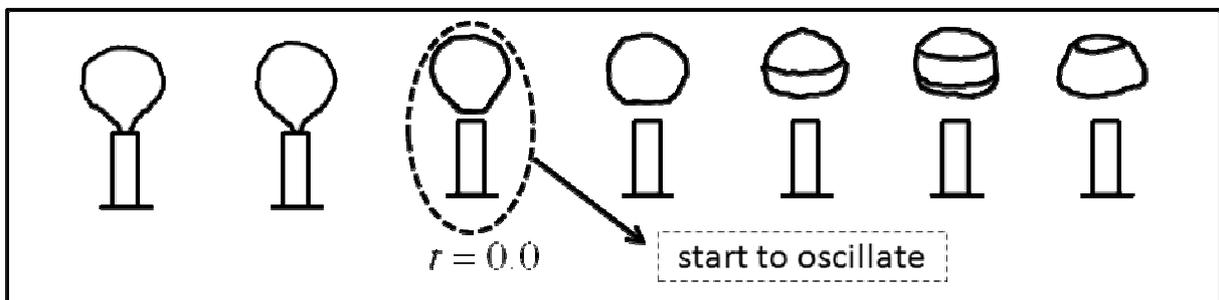


Fig.3. Bubble formation as a function of time [24].

Strasberg [3] showed that an ellipsoid bubble has a central frequency that is greater than for a sphere with the same volume, and this frequency approaches that of a sphere as the ellipse eccentricity decreases. The central frequency of a spherical bubble excited by an external source at a depth  $h$  close to the sea surface is given by [16]

$$f_h = \frac{f_0}{\sqrt{1 - \frac{r_0}{2h} - \left(\frac{r_0}{2h}\right)^4}} \quad (7)$$

Pumphrey and Williams [16] demonstrated that the spectra of the pressure signal generated by an oscillatory bubble decays in time but its central frequency remains constant. This is

illustrated in Fig. 4 where the spectrum of a rising bubble is shown when the bubble leaves the nozzle, and a short time later when it is still oscillating. This figure shows that there is in fact a slight variation in the central frequency. This can be attributed to pressure variations in the laboratory test water tank as the bubble raises. A decrease in the spectrum amplitude and its width may be attributed to the signal being analyzed in different observation windows.

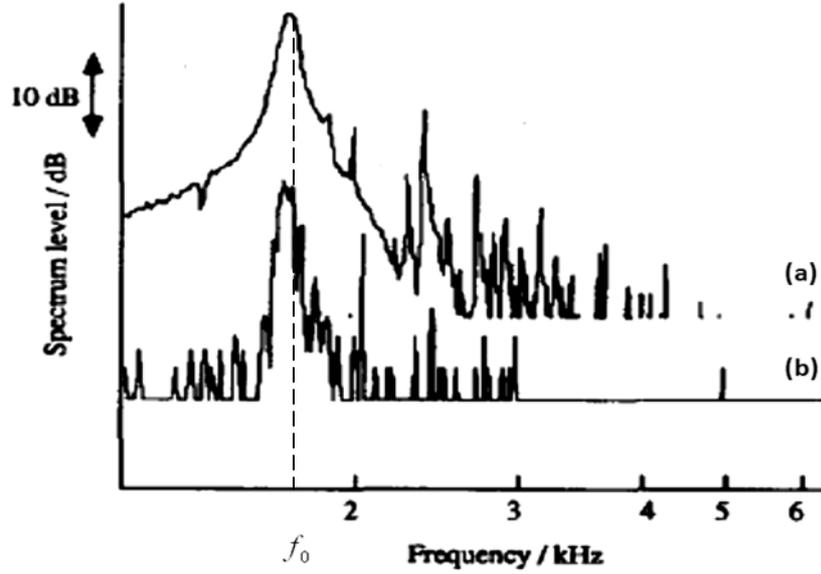


Fig.4. Spectra of bubbles (a) leaving a nozzle and (b) a short time later [15].

## 2. DYNAMIC MODEL

The dynamics of a single bubble in mode  $n=0$  can be described by a second-order linear differential equation [14] corresponding to a damped spring-mass model. The surrounding bubble fluid behaves like a mass that periodically expands and contracts the air in the bubble based on the spring characteristics. The displacement  $x(t)$  and angular oscillation frequency  $\omega_0 = 2\pi f_0$  are given by

$$x(t) = Ae^{-2\beta\pi f_0 t} \sin(\omega_0 t) \quad (8)$$

and

$$\omega_0 = (k/m)^{\frac{1}{2}} \quad (9)$$

where  $m$  is the displaced water mass,  $k$  is the spring stiffness due to the air in the bubble and  $\beta$  is the damping coefficient. The stiffness  $k$  of an air bubble can be approximated as [14]

$$k_a \approx 12\pi\nu r_0 P_0 \quad (10)$$

where  $r_0$  is bubble radius and  $\nu$  is the polytropic exponent.

### 3. DAMPING

The exponential damping of the bubble oscillations can be attributed to shear viscosity, acoustic radiation, and thermal conductivity [6,16]. The damping coefficient is given by

$$\beta = \beta_v + \beta_r + \beta_{th} \quad (11)$$

where  $\beta_v$ ,  $\beta_r$  and  $\beta_{th}$  are the viscous, radiation and thermal damping coefficients, respectively. Thermal damping has been investigated in [4, 14]. The thermal damping coefficient  $\beta_{th}$  for frequencies below 10 kHz is given by

$$\beta_{th} = \frac{3(\gamma - 1)}{2r_0} \sqrt{\frac{\omega_0 D}{2}} \quad (12)$$

where  $D$  is the thermal diffusivity of air which is approximately  $D \cong 0.2 \text{ cm}^2 / \text{ s}$  for air bubbles,  $r_0$  is the mean radius and  $\omega_0$  is the natural angular frequency. Some of the bubble energy is dissipated by the shape radiation (oscillations). The radiation damping coefficient was first calculated by Smith [13] as

$$\beta_r = \frac{\omega_0^2 r_0}{2c} = \frac{\omega_0 \sqrt{3\gamma P_0}}{2c} \quad (13)$$

where  $P_0$  is the ambient pressure and  $c$  is the sound velocity in water. Viscous damping was first investigated by Mallock [17] and later by Poritsky and Devin [14]. The viscous damping coefficient is given by

$$\beta_v = \frac{2\nu}{r_0^2} = \frac{2\nu\omega_0^2}{3\gamma P_0} \quad (14)$$

where  $\nu \cong 0.01 \text{ cm}^2 / \text{ s}$  is the water kinematic viscosity. Fig. 5 shows the viscous, thermal and radiation damping coefficients, as well as the total damping, as a function of the air bubble central frequency  $f_0$  for bubbles radii from 3  $\mu\text{m}$  to 3mm.

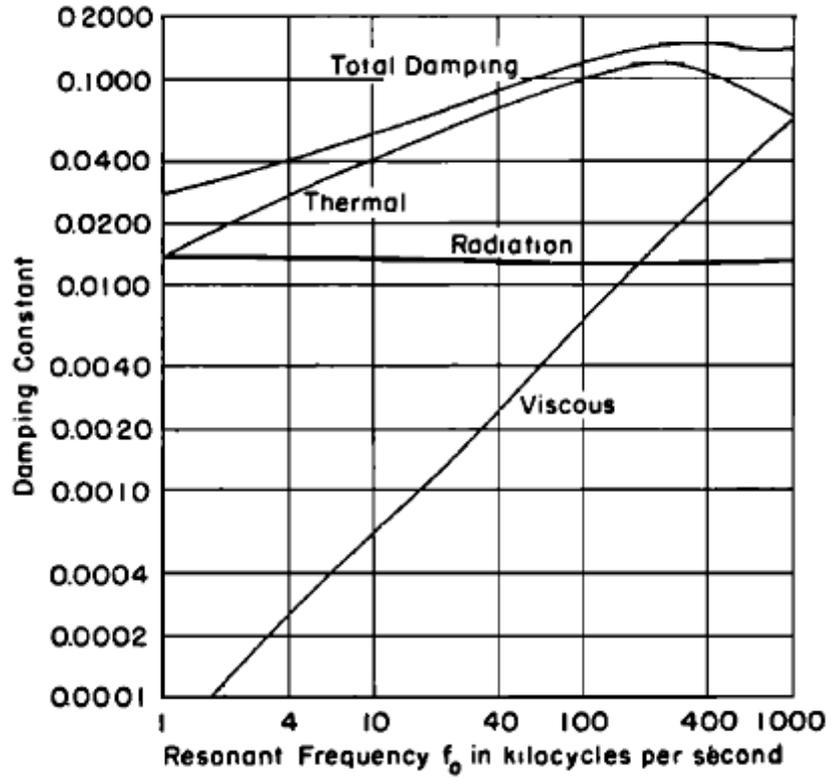


Fig.5. Thermal, radiation and viscous damping coefficients, and the total damping, for air bubbles in water [14].

It has been shown experimentally that the oscillations decay faster than that predicted by the theoretical models [9,19]. Medwin and Beaky [16] have extensively investigated the oscillatory behavior of bubbles and the corresponding damping behavior.

#### 4. SINGLE BUBBLE ASCENT VELOCITY

Once generated, a bubble ascends with a given velocity. An excellent survey of the bubble rise velocity  $v(t)$  is given in [10]. This velocity has been characterized via two closed form expressions. Assuming that the water has greater viscosity and density compared to the bubble, Hadmard and Rybczynski [18] obtained the following expression

$$v(t) = \frac{\rho g r^2(t)}{3\mu} \quad (15)$$

where  $r(t)$  is the bubble radius that is a function of the nozzle diameter, ambient pressure, liquid viscosity  $\mu$ , acceleration due to gravity  $g$ , and liquid density  $\rho$ . A second expression obtained by Stokes [18] is frequently used for bubbles with “dirty” surfaces (the internal air circulation is ignored), and is given by

$$v_{st}(t) = \frac{2\rho g r^2(t)}{9\mu} \quad (16)$$

These expressions have been evaluated for small bubbles (with Reynolds number less than 1). No closed form expressions have been derived for the ascent velocity of larger bubbles. Based on experimental results, empirical expressions for the velocity of larger bubbles have been obtained [10]. The bubble velocity  $v(t)$  given by (15) is used in this paper to evaluate the distance a bubble travels upward while it is oscillating.

## 5. BUBBLE GENERATED BY LINEAR NUZZLE ARRAY

A single bubble generates a pressure signal that is too small for use in remote sensing. It will be demonstrated later that a larger number of oscillating bubbles will, under certain conditions, increase the total pressure generated. By providing a sufficient air supply one can generate a dense (one bubble following immediately after the previous bubble), train of bubbles. Such a vertically ascending train will contain  $n$  active bubbles (starting from the nozzle) that generate an acoustic signal. This is preceded by a train of passive bubbles that have ceased to oscillate. The number of active bubbles depends on the signal duration  $\tau$  of a single bubble. The thickness of the layer of active bubbles is determined by their ascent velocity as given by (14) or (15). Generalizing the above idea to a linear array of  $N$  identical nozzles, each generating  $n$  active bubbles, the total number of active bubbles is  $Nn$ . This array concept is illustrated in Fig. 7.

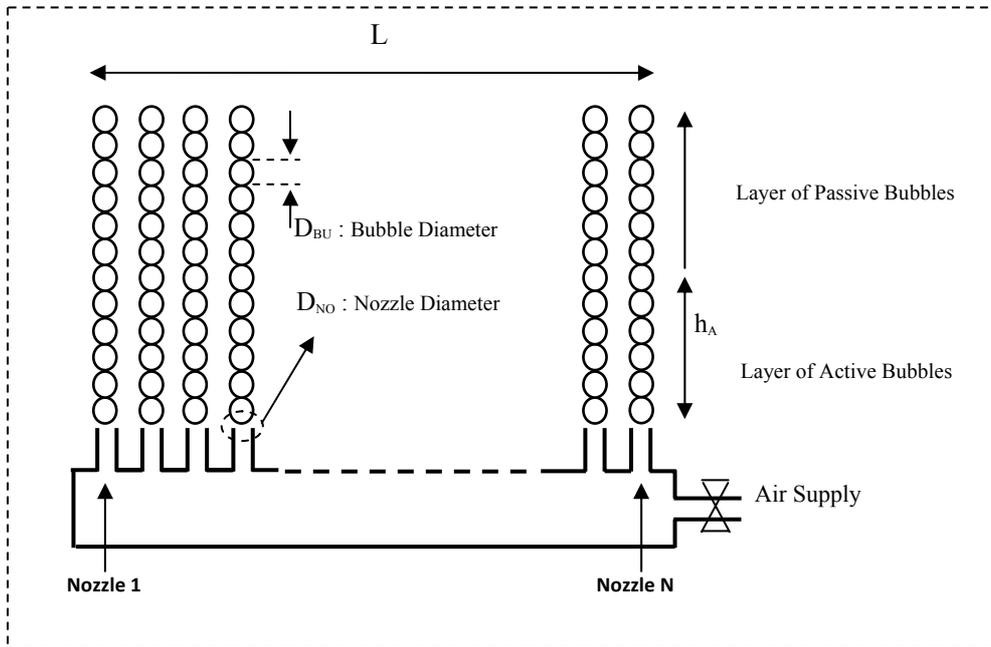


Fig.7. Bubble curtain produced by  $N$  nozzles.

In order to minimize the physical length of the array  $L$  it is further assumed that the nozzles with diameter  $D_{NO}$  are separated by a distance equal to the bubble diameter  $D_{BU}$ . This will form a dense curtain of bubbles with each bubble touching its immediate neighbors. The properties of a dense bubble curtain generated by two linear arrays with  $N=20$  and  $N=100$

nozzles for four different nozzle diameters are shown in Table 1. The exact relationship between the nozzle and bubble diameters is not found in the literature. Therefore experimental results from [24] were used in Table 1. Alternatively the frequency of oscillation can be also calculated using (4). Damping coefficient  $\beta$  was obtained from Fig. 5

Tab.1. Bubble properties for different nozzle arrays.

$D_{NO}$ [mm]	$D_{BU}$ [mm]	$f_0$ [kHz]	$\beta$ [ms <sup>-1</sup> ]	$v_0$ [mm/ms]	$\tau$ [ms]	$h_A$ [cm]	Total number of active bubbles: $nN$ [in thousands]		$L$ [cm] for $N=100$
							N=20	N=100	
2	6.4	1.33	26	8.2	88.5	72.6	2.26	11.3	64
1	4	1.53	30	3.2	76.7	24.6	1.22	6.1	40
0.6	3	1.78	33	1.8	69.8	12.6	0.82	4.1	30
0.2	2	2.49	37	0.8	62.2	5.0	0.48	2.4	20

Table 1 shows that the minimum length for  $N=100$  varies between 20 to 64 cm which is a reasonable length for practical applications. Due to the very small time delays between all active bubbles in a column, the total pressure generated by these bubbles tends to combine constructively and increases with the number of active bubbles. The simulation result for bubbles using 1 mm diameter nozzles with parameters as for Table 1 is shown in Fig. 8. Each nozzle produces a dense column of  $n$  bubbles that behaves like a single source due to the very small time delay between the signals generated. The resulting signal amplitude will depend on the total number of active bubbles  $nN$ . The upper bound on of the increase in pressure level is  $20\log_{10}(nN)$  [dBs]

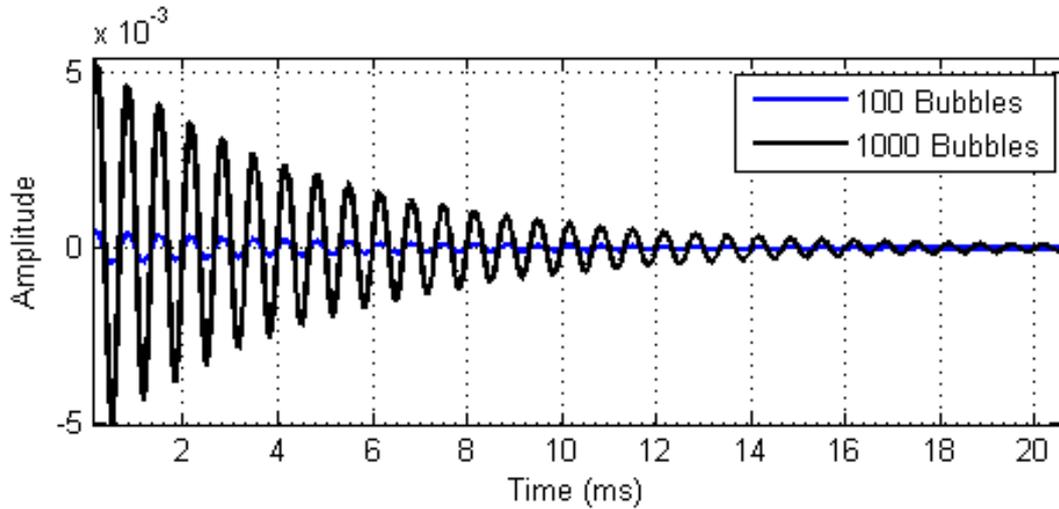


Fig.8. Received signals for 100 and 1000 oscillating bubbles.

## 6. CONCLUSIONS AND FUTURE WORK

Various properties of bubbles were surveyed and investigated in view of potential applications for passive remote sensing. The bubbles may be formed in a controlled way by an array of nozzles, or in an uncontrolled way by a leaking underwater gas line as shown in Fig. 9 or by natural sources. For controlled bubbles, one can investigate tracing the position and depth of a diver wearing a suitable bubble generating nozzle array such as a short vertical pipe or a horizontal ring with a multitude of small holes acting as nozzles. The exhaled air will periodically activate the array. The array depth can be determined using the dependence of signal frequency on depth as given by (4). The acoustic signal strength, frequency and radiation pattern can be controlled by the distribution and diameter of the nozzles forming the array. In an uncontrolled scenario, new models considering random bubble sizes and distributions can be developed together with a methodology for signal reception and classification.

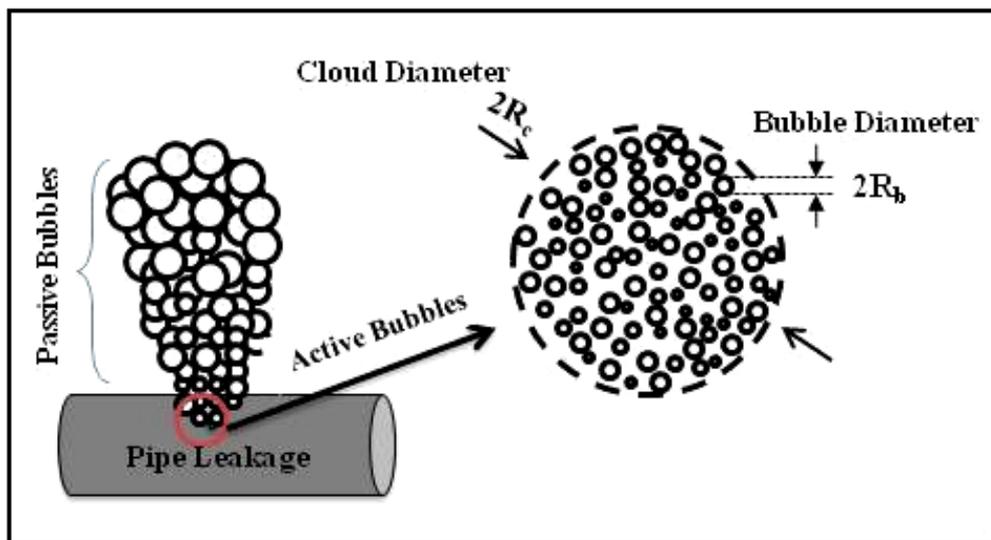


Fig.9. Continuous bubble leakage and bubble cloud scenario.

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