MODELING OF VERTICAL DISTRIBUTION OF SOUND SPEED IN WATER USING BEZIER COURVES

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Modeling of the vertical distribution of the sound speed in water is essential problem of hydroacoustics. There are many mathematics methods of modeling one variable functions. In the paper modeling of the one variable function for the vertical distribution of the sound speed in water using Bezier functions have been shown.

INTRODUCTION

Knowledge about vertical distribution of the sound speed in water is essential issue in theory of acoustic wave's propagation [1, 3], determination of the depth using acoustic methods [3, 7, 8], determination of measurement's accuracy [10] and determination of acoustic wave reflection points in bathymetric surveys [3].

Many methods were been used for describing the vertical distribution of the sound speed in water [4, 5, 6, 7, 10, 11, 12, 13, 14], e.g. Uniform B-Splines, NonUniform Rational B-Splines NURBS and other well known interpolation methods [2].

1. BEZIER CURVES

Let's choose in free method a sequence of n+1 points $p_0,..., p_n$ and let's into consideration a broken line with these points. Now, we divide all n segments of his broken line in established proportion. This proportion can be described by one number parameter t: each section is divided in proportion: $t:1-t$. Next, we receive n points, which are points of another broken line, which consists of $n-1$ sections. This process is repeated for obtaining one point. One of Bezier curve definition described it as p curve, when each point of $p(t)$ can be constructed using adequate t.

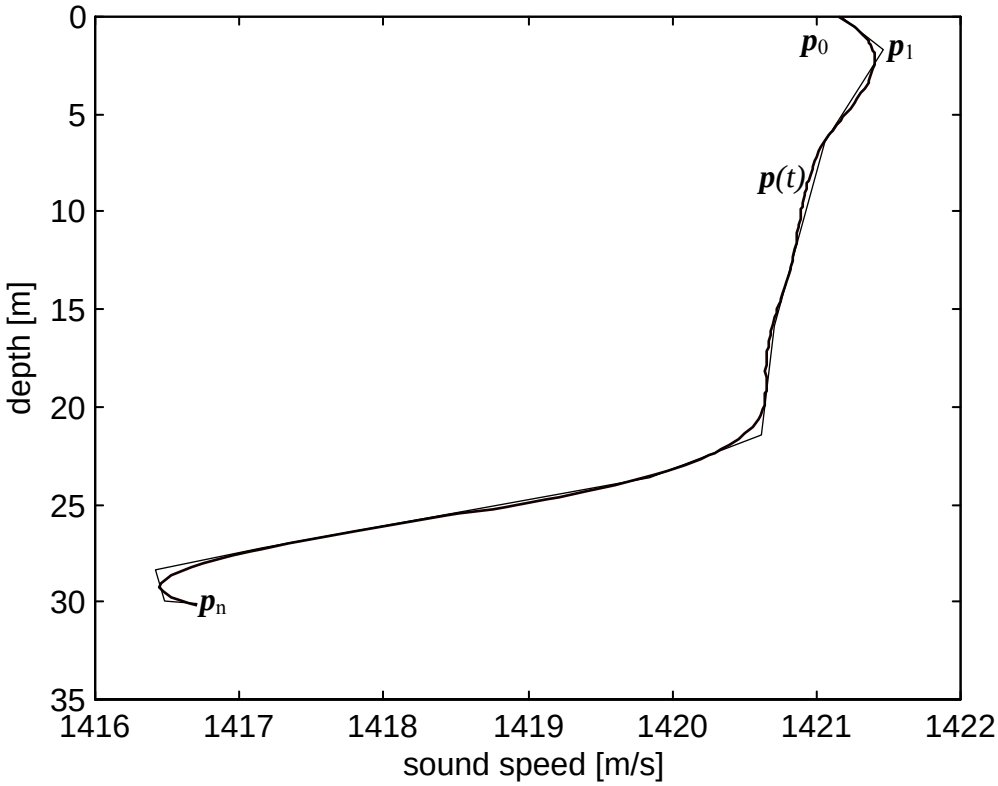

Fig.1. De Casteljau algorithm of vertical distribution of sound speed in water

Described algorithm is called Casteljau algorithm, when for $t \in [0,1]$ corners are cut. As a result of this process, the broken line makes the curve. The iteration step can be written in the form [1]:

$$p_i^{(j)} := (1-t)\,p_i^{(j-1)} + t p_{i+1}^{(j-1)} \quad \text{for} \quad i = 0,\ldots,n \quad \text{and} \quad j = 1,\ldots,n \tag{1}$$

Start points are called control points, the output broken lines called Bezier control line. Looking for de Casteljau algorithm we can observe:

- Bezier curve is polynomial one: if there are n+1 control points, curve's coordinates are described by polynomials of t variable of the degree not higher than n: so Bezier curve term is specific for individual polynomial curve representation;
- the curve has the convex property: for $t \in [0,1]$ a point $p(t)$ lies on convex line of $p_0,\ldots,p_n$ points;
- construction of the curve is affine constant: the picture of $p_0,\ldots,p_n$ points in free affine transformation determines the picture of the p curve in this transformation;
- occurs the interpolation of final points of the broken line: $p(0) = p_0$, $p(1) = p_n$.

2. BERNSTEIN POLYNOMIALS

Bernstein polynomials of n – degree are defined by equation [1]:

$$B_i^n(t) \overset{def}{=} \binom{n}{i} t^i (1-t)^{n-i} \quad \text{for } i = 0,\ldots,\text{n}. \tag{2}$$

These polynomials are linear independent. They determine the space base of polynomials of degree not more than n, because they are n+1.

$$B_i^n(t) \overset{def}{=} 0 \quad \text{for } i < 0 \text{ or } i > 1. \tag{3}$$

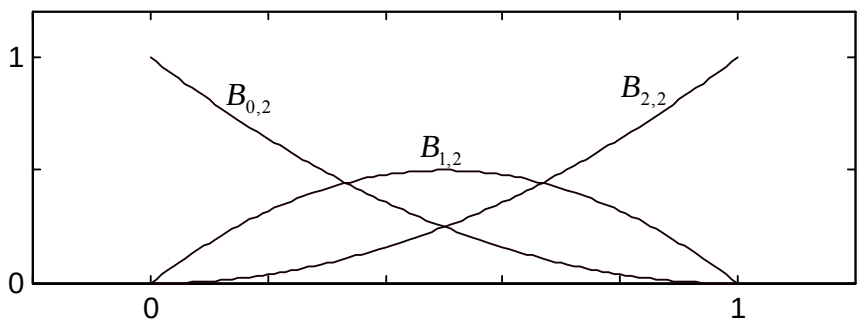

Fig.2. Graphs of Bernstein polynomials

Bernstein polynomials meet recurrent relationship [1]:

$$B_i^n(t) = (1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t). \tag{4}$$

Polynomial $B_0^0(t)$ is equal to 1. For each n we also have $\binom{n}{0} = \binom{n}{n} = 1$, so for i = 0 and i = n foregoing equation results from an agreement:

For $n > 1$, $i = 1,\ldots,n$-1

$$(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t). \tag{5}$$

Polynomials of higher and higher degrees can be obtained using the pattern, which is the generalization of Pascal triangle.

Turned out, that control points of Bezier curve are coefficients of the curve in Bernstein polynomials space:

$$p(t) = \sum_{i=0}^{n} p_i B_i^n(t). \tag{6}$$

3. DEGREE ELEVATION

Calculating

$$p(t)=\sum_{i=0}^{n} p_i B_i^n(t)=t\sum_{i=0}^{n} p_i B_i^n(t)+(1-t)\sum_{i=0}^{n} p_i B_i^n(t)=\sum_{i=0}^{n+1} \bar{p}_i B_i^{n+1}(t)\,, \tag{7}$$

we obtain for $i = 0,\ldots,n+1$

$$\bar{p}_i=\frac{n+1-i}{n+1}\,p_i+\frac{i}{n+1}\,p_{i-1}\,. \tag{8}$$

Obtained equation is well described, because in relationships for $\bar{p}_0$ and $\bar{p}_{n+1}$ indefinite points p_{-1}, p_{n+1} are multiply by 0. In this way coefficients (control points) of output curve corresponding to Bernstein polynomial base of $n + 1$ degree have been obtained. This process is called degree elevation.

Degree elevation can be used: for obtaining the freedom (the number of control points increases), for agreeing the representation of curves (curves joining, data export), in theoretic considerations.

4. RESULTS

Measurements of the vertical distribution of the sound speed in water have been done during hydrograprhic surveys of the Slupsk Bank in March, 2010 on the hydrographic vessel OH266. The area of the surveys has been shown below.

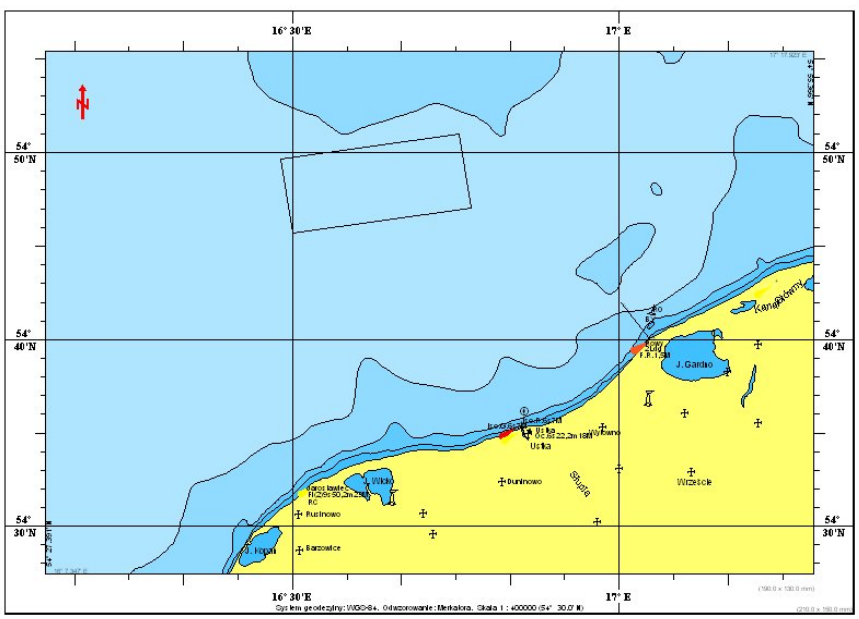

Fig.3. Hydrographic area of the Slupsk Bank

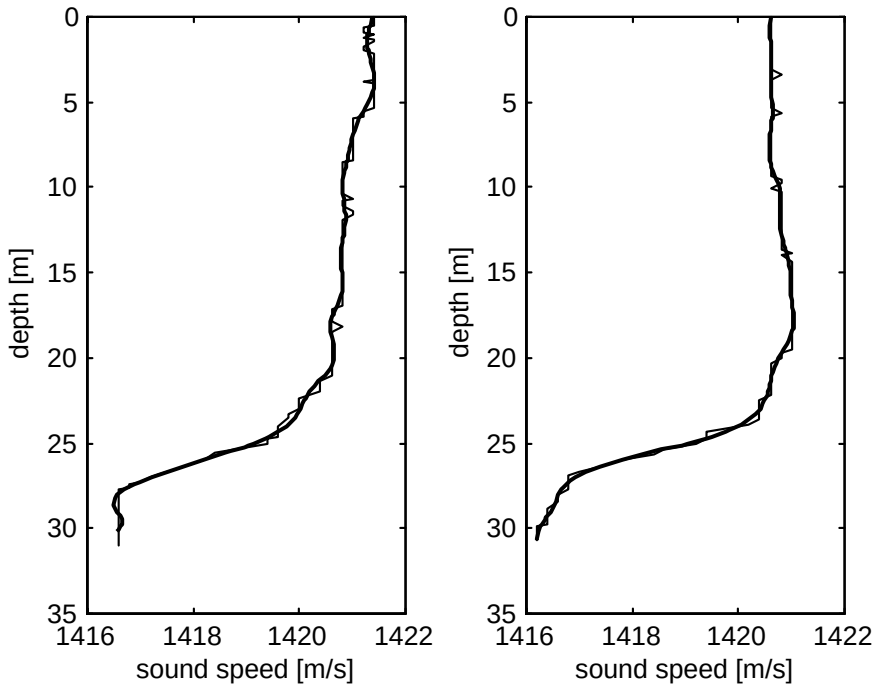

Fig.4. Real and approximated vertical distributions of the sound speed in water – 25-th of March, 2010: 12:00 and 20:00

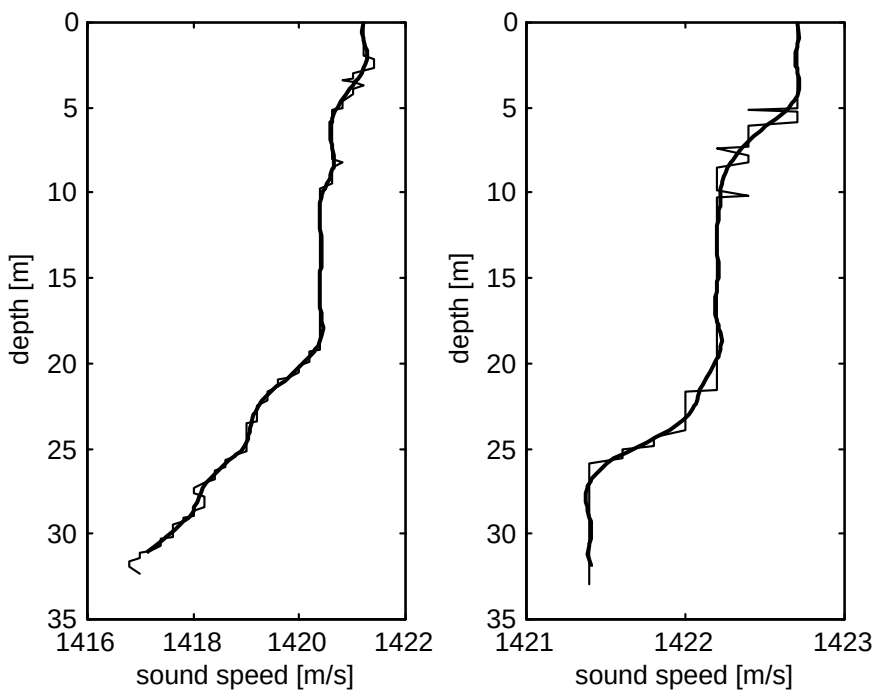

Fig.5. Real and approximated vertical distributions of the sound speed in water – 26-th and 27-th of March, 2010

5. CONCLUSIONS

Approximation of the nonlinear one variable function is usable in hydroacoustics for modeling vertical distribution of the sound speed in water. Used algorithms make possible to determinate the depth as the function of the sound speed in water, and also modeling the trajectory of the acoustic ray as the result of the refraction

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