THE ACOSTIC RADIATION IMPEDANCE OF A CIRCULAR MEMBRANE VIBRATING NEAR THE THREE-WALL CORNER

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A flat circular membrane is located near the three- wall corner, limited by the three rigid baffles arranged perpendicularly to each other. The problem of sound radiation has been solved using the spectral form of the Green function for this Neumann boundary value problem together with the complete eigenfunction system of the axisymmetric and asymmetric modes of the membrane is excited by a surface vibrating harmonically with respect to time within the vacuum. The membrane is excited by a surface force. The acoustic attenuation 3399effect has been taken into account as well as the influence of the corner baffles. The resultant sound pressure and the resultant acoustic impedance have been presented as their eigenfunction series. The modal, self and mutual, radiation resistance has been presented in the form of the approximation valid within the low frequency vibration range. The low frequency approximation for the modal radiation reactance has been obtained on the basis of the radiation resistance using the Hilbert transform.

INTRODUCTION

A number of detailed research reports on the problems of sound radiation of vibrating surface sources embedded into a flat rigid baffle was presented so far.

The authors of this paper have extended this area research with the energy aspect of the pistons radiating the acoustic waves into the subspaces of the two wall corner and the three wall corner [1, 2]. The basis of the analysis has been the spectral form of the Green function satisfying the Neumann boundary conditions at the two wall corner and the three wall corner. This Green function has been used to obtain the modal impedance [3], especially within the low frequency range. Further, this modal impedance has been used to present the total sound power radiated by an asymmetrically vibrating circular membrane located in the vicinity of the two wall corner.

This paper contains a continuation of the previous research and focuses on using the spectral form of the Green function for the Neumann boundary value problem at the subspace of the three wall corner and to obtain the modal radiation impedance of a vibrating circular membrane embedded into one of the three rigid baffles situated perpendicularly to each other. The total sound power radiated has also been focused on. The membrane has been excited by an asymmetric surface force. The acoustic attenuation has been included in the results presented herein.

Using the modal analysis has led to the formulation of the modal radiation impedance in Eq. (3.10), and to the approximation of the modal radiation resistance in Eq. (4.5) valid when the linear sizes of the membrane are small as compared with the radiated wave length, i.e. when $k_0 a = 2\pi a/\lambda << 1$. Further, the Hilbert transform has been used to obtain the modal radiation reactance of the two initial axisymmetric modes in Eq. (4.9) directly from the corresponding modal radiation resistance from Eq. (4.6). Some sample numerical results have been illustrated in Figs. 2–6. The results presented herein can be used for some further numerical analyses of the total sound power radiated by the membrane excited by various axisymmetric and asymmetric surface forces.

1. THE SOUND PRESSURE

A circular membrane of radius a vibrates axisymmetrically within the subspace of the three wall corner bounded by the three rigid baffles situated perpendicularly each other. The subspace $0 \leq x \leq \infty$, $0 \leq y \leq \infty$, $0 \leq z \leq \infty$ is filled with the lossless gaseous medium of rest density ρ_0. The membrane is embedded into the baffle z=0 and is illustrated in Fig. 1.

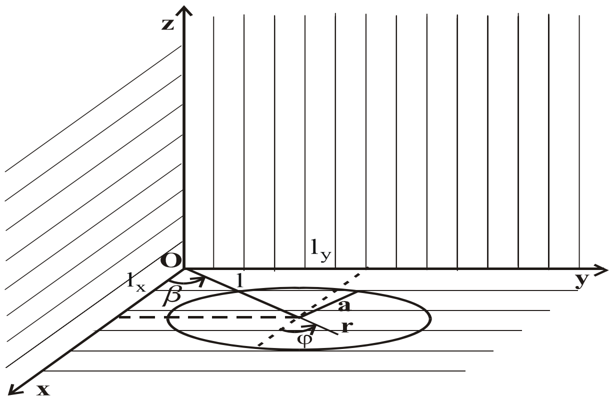

Fig.1 The location of the membrane at the three-wall corner

The sound pressure radiated by the membrane $p(\vec{r},t)$ can be formulated as $p(\vec{r},t) = p(\vec{r})e^{-i\omega t}$ for some time harmonic processes where ω is the circular frequency. The sound pressure amplitude has been defined using the Green function

$$p(\vec{r}) = -ik_0\rho_0 c \int_{S_0} v(\vec{r}_0)G(\vec{r}_0 \mid \vec{r})dS_0, \tag{2.1}$$

where $k_0 = 2\pi/\lambda$ is the acoustic wavenumber, λ is the wavelength and c is the sound velocity within the gaseous medium under consideration. The vibration velocity amplitude of the membrane is $v(\vec{r}_0) = v(r_0,\rho_0) = -i\omega W(r_0,\rho_0)$. The Green function for the Helmholtz equation satisfying the Neumann boundary conditions at the baffles x=0, y=0 and z=0 as well as "the sharpened Sommerfeld radiation condition" [4] has been expressed in its Fourier representation [2]

$$G(\vec{r}\,|\,\vec{r}_0) \equiv G(x,y,z\,|\,x_0,y_0,0) = \frac{4i}{\pi^2}\int_{\zeta=0}^{+\infty}\int_{\eta=0}^{+\infty} e^{i\gamma z}\cos(\xi x_0)\cos(\xi x)\cos(\eta y_0)\cos(\eta y)\,d\xi d\eta, \tag{2.2}$$

where $\gamma = \sqrt{k_0^2 - \zeta^2 - \eta^2}$ and remembering that the sound source is located in the half-plane $z_0 = 0$.

It is enough to obtain the sound pressure for z=0 while analyzing the energy aspect of the sound radiation, i.e. the acoustic impedance. The Cartesian coordinates of the field point and the source point can be converted to their local polar coordinates for $z = z_0 = 0$

$$\begin{aligned} &x = l_x + r\cos\varphi, \qquad y = l_y + r\sin\varphi, \\ &x_0 = l_x + r_0\cos\varphi_0, \quad y_0 = l_y + r_0\sin\varphi_0, \end{aligned} \tag{2.3}$$

where $l_x = l\cos\beta$ and $l_y = l\sin\beta$ are the Cartesian coordinates of the central point of the membrane (Fig. 1).

The transverse deflection amplitude of the membrane has been formulated as the following double eigenvalue series

$$W(r,\varphi) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} c_{mn} W_{mn}(r,\varphi), \tag{2.4}$$

where the eigenfunction of the mode (m,n) is (cf. Eq. (B.3))

$$W_{mn}(r,\varphi) = \sqrt{\varepsilon_m}\,\frac{J_m(k_{mn}r)}{J_{m+1}(\beta_{mn})}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}, \tag{2.5}$$

and $\varepsilon_0 = 1$, $\varepsilon_m = 2$ for $m \ge 1$, $\beta_{mn} = k_{mn}a$ is the root of the membrane's frequency equation $J_m(\beta_{mn}) = 0$. The characteristic radiation function has been formulated as

$$M(\zeta,\eta) = \int_0^a\int_0^{2\pi} v(r_0,\varphi_0)\cos[\xi(l_x + r_0\cos\varphi_0)]\cos[\eta(l_y + r_0\sin\varphi_0)]\,r_0 dr_0 d\varphi_0, \tag{2.6}$$

Applying Eq. (2.4), (2.5), (2.1), (2.2), (2.6) leads to the sound pressure amplitude in the form of

$$p(r,\varphi) = (2/\pi)^2\rho_0 c k_0 \int_0^{+\infty}\int_0^{+\infty} M_{mn}(\zeta,\eta)\cos(\xi x)\cos(\eta y)\frac{d\zeta d\eta}{\gamma}, \tag{2.7}$$

where

$$M(\zeta,\eta) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} c_{mn} M_{mn}(\zeta,\eta), \tag{2.8}$$

$$M_{mn}(\zeta,\eta) = \pi a^2(-i\omega)i^m \frac{\sqrt{\varepsilon_m}\beta_{mn}J_m(\tau a)}{\beta_{mn}^2 - (\tau a)^2} \times$$
$$\times \begin{Bmatrix} \cos(m\alpha)\cos(\eta l_y) \\ -i\sin(m\alpha)\sin(\eta l_y) \end{Bmatrix} \begin{Bmatrix} +[1+(-1)^m]\cos(\zeta l_x) + i[1-(-1)^m]\sin(\zeta l_x) \\ -[1+(-1)^m]\sin(\zeta l_x) + i[1-(-1)^m]\cos(\zeta l_x) \end{Bmatrix}, \quad (2.9)$$

and $\tau = \sqrt{\zeta^2 + \eta^2}$, $\zeta = \tau\cos\alpha$, $\eta = \tau\sin\alpha$, $d\zeta d\eta = \tau d\tau d\alpha$. The integration in Eq. (2.7) is performed over the variable $\tau = \tau' + i\tau''$ along the real axis within the limits $(0,\infty)$ whereas the integration over the real variable α is performed within the limits $(0,\pi/2)$.

2. SOUND RADIATION IMPEDANCE

After using the sound pressure amplitude from Eq. (2.1) the mutual sound power of the vibration modes (m,n) and (m',n') has been formulated as [3]

$$\Pi_{mn,m'n'} = -\frac{ik_0\rho_0 c}{2}\int_S\int_{S_0} v_{mn}(\vec{r}_0)G(\vec{r}_0\,|\,\vec{r})v^*_{m'n'}(\vec{r})dS_0\,dS, \quad (3.1)$$

whereas the normalized radiation impedance has been formulated as

$$\zeta_{mn,m'n'} = \frac{\Pi_{mn,m'n'}}{\rho_0 cS\sqrt{\langle|v_{mn}|^2\rangle\langle|v_{m'n'}|^2\rangle}}, \quad (3.2)$$

where

$$\langle|v_{mn}|^2\rangle = \frac{1}{2S}\int_S v_{mn}(\vec{r})\,v^*_{mn}(\vec{r})\,dS \quad (3.3)$$

is the time-averaged and surface-averaged vibration velocity square of the mode (m,n) and is equal to $(1/2)\omega^2$ for any mode. The total sound power is [3]

$$\Pi = \sum_{m=0}^{\infty}\sum_{m'=0}^{\infty}\sum_{n=1}^{\infty}\sum_{n'=1}^{\infty} c_{mn}c^*_{m'n'}\Pi_{mn,m'n'}. \quad (3.4)$$

and it is necessary to determine the coefficients c_{mn} as well as the modal sound power radiated $\Pi_{mn,m'n'} = \Pi'_{mn,m'n'} - i\Pi''_{mn,m'n'}$, active and reactive. The equation of motion of the membrane excited by a surface force, including the acoustic attenuation, was analysed earlier in [3] and also has been presented in Appendix B for convenience. This appendix contains also the set of algebraic equations (B.5) together with its solutions c_{mn} as well as the total sound power from Eq. (B.7) in a form different than in the Eq. (3.4).

For the purpose of the later obtaining the values of the coefficients c_{mn} it is necessary former to obtain the modal radiation impedance. This modal radiation impedance has been formulated using the characteristic functions from Eq. (2.8). The expressions in Eqs. (2.1) and (2.7) have been multiplied side by side by $v^*(\vec{r}) = \sum_{m'}^{\infty}\sum_{n'}^{\infty} c^*_{m'n'}v^*_{m'n'}(\vec{r})$, integrated over the membrane surface and the general terms of the quadruple series have been compared each other. Eqs. (2.2), (3.1) and (3.2) have been used giving

$$\zeta_{mn,m'n'} = \left(\frac{2}{\pi}\right)^2 \frac{k_0}{S\omega^2} \int_0^{+\infty} \int_0^{\pi/2} M_{mn}(\tau,\alpha)\, M^*_{m'n'}(\tau,\alpha) \frac{\tau\, d\tau d\alpha}{\sqrt{k_0^2-\tau^2}}, \tag{3.5}$$

where the integration has been performed along the real axis $O\tau'$ of the complex variable $\tau=\tau'+i\tau''$ omitting the branch point $\tau'=k_0$.

Eq. (2.11) represents the characteristic function $M_{mn}(\tau,\alpha)$ in its elementary form which used in the integrand in Eq. (3.5) makes it possible to compute the integral over the variable α within the limits $(0,\pi/2)$.

The products of the two different characteristic functions (cf. Eq. (2.11)) can be formulated as

$$M_{mn}(\tau,\alpha)M^*{}_{m'n'}(\tau,\alpha) = A_{mm'}\Psi_{mn}(\tau)\Psi_{m'n'}(\tau)V_{mm'}(\tau,\alpha), \tag{3.6}$$

with the following denotations

$$A_{mm'} = (\omega S)^2 i^{m+m'}\sqrt{\varepsilon_m\varepsilon_{m'}}\,,\ \Psi_{mn}(\tau) = \frac{\beta_{mn} J_m(\tau a)}{\beta_{mn}^2 - (\tau a)^2},$$

$$\begin{aligned} V_{mm'}(\tau,\alpha) = &\begin{Bmatrix}\cos(m\alpha)\cos(m'\alpha)\\ \sin(m\alpha)\sin(m'\alpha)\end{Bmatrix}\\ &\times\{\, q_1\left[(-1)^{m'}(1\pm\cos(2\tau\, l_y \sin\alpha)) \pm \cos(2\tau\, l_x\cos\alpha) + \cos(2\tau\, l_x\cos\alpha)\cos(2\tau\, l_y\sin\alpha)\right],\\ &+ iq_2\left[\pm\sin(2\tau\, l_x\cos\alpha) + \sin(2\tau\, l_x\cos\alpha)\cos(2\tau\, l_y\sin\alpha)\right]\} \end{aligned} \tag{3.7}$$

where

$$q_1 = \frac{1}{2}\left[1+(-1)^{m+m'}\right],\ q_2 = \frac{1}{2}\left[1-(-1)^{m+m'}\right]. \tag{3.8}$$

The modenumbers m and m' can be even or odd. If m is even and m’ is odd or m is odd and m’ is even then m+m’ and m-m’ are odd and $q_1 = 0$, $q_2 = 1$. Another case is when m and m’ are even or m and m’ are odd. In this case m+m’ and m-m’ are even and $q_1 = 1$, $q_2 = 0$. At this stage of analysis we can already conclude that for taking into consideration influences of all three rigid baffles on radiating the sound pressure it is necessary to accept that $q_1 = 1$, $q_2 = 0$. The result of integrating is the following ((A.1) – (A.6))

$$\begin{aligned} \frac{4}{\pi}\int_0^{\pi/2} V_{mm'}(\tau,\alpha)d\alpha = &\, q_1\Big\{(-1)^{m'}\left[\frac{2}{\varepsilon_m}\delta_{mm'} + J_{m+m'}(2\tau\, l_y)\right.\\ &\left.\pm J_{m-m'}(2\tau\, l_y)\right] + i^{m+m'}\left[J_{m+m'}(2\tau\, l_x) \pm (-1)^{m'} J_{m-m'}(2\tau\, l_x)\right.\\ &\left.\pm\cos(m+m')\beta J_{m+m'}(2\tau\, l) + (-1)^{m'}\cos(m-m')\beta\right] J_{m-m'}(2\tau\, l)\Big]\Big\},\\ &+ q_2 i^{m+m'}\left[J_{m+m'}(2\tau\, l_x) \pm (-1)^{m'} J_{m-m'}(2\tau\, l_x)\right.\\ &\left.\pm\cos(m+m')\beta J_{m+m'}(2\tau\, l) + (-1)^{m'}\cos(m-m')\beta J_{m-m'}(2\tau\, l)\right] \end{aligned} \tag{3.9}$$

where in the symbol $\pm$ the plus sign (here upper) concerns the excitation $\cos m\varphi$ for m=0,1,2,…, in Eq. (2.5) whereas the negative sign (here bottom) concerns the excitation $\sin m\varphi$ for m=1,2, …

The modal mutual radiation impedance has been formulated as a single integral

$$\xi_{mn,m'n'} = \frac{k_0 S}{\pi}\sqrt{\varepsilon_m \varepsilon_{m'}} \int_0^\infty \{ q_1(V_1+V_2+V_3) + q_2(V_2+V_3)\} \Psi_{mn}(\tau)\Psi_{m'n'}(\tau) \frac{\tau\, d\tau}{\sqrt{k_0^2-\tau^2}}, \qquad (3.10)$$

with the following denotations

$$V_1 = (-1)^{m'} i^{m+m'} \left[\frac{2}{\varepsilon_m}\delta_{mm'} + J_{m+m'}(2\tau\, l_y) \pm J_{m-m'}(2\tau\, l_y) \right],$$

$$V_2 = (-1)^{m+m'} J_{m+m'}(2\tau\, l_x) \pm (-1)^m J_{m-m'}(2\tau\, l_x), \qquad (3.11)$$

$$V_3 = \pm(-1)^{m+m'} \cos(m+m')\beta J_{m+m'}(2\tau\, l) + (-1)^m \cos(m-m')\beta J_{m-m'}(2\tau\, l).$$

The square root $\sqrt{k_0^2-\tau^2}$ assumes its real values for $0 \le \tau \le k_0$ whereas for $k_0 \le \tau < \infty$ it assumes its imaginary values $i\sqrt{\tau^2-k_0^2}$. The remaining part of the integrand assumes its real values within the integration limits $0 \le \tau < \infty$. For this reason, the modal radiation impedance $\xi_{mn,m'n'}$ has been formulated using the modal radiation resistance $\theta_{mn.m'n'}$ and the modal radiation reactance $\chi_{mn.m'n'}$ as follows $\xi_{mn,m'n'} = \theta_{mn,m'n'} - i\chi_{mn,m'n'}$.

In the specific case, when the liming transition is realized $l_x \to \infty$ within the integrand of Eq. (3.11) the expression is obtained

$$\lim_{l_x\to\infty} \xi_{mn,m'n'}(l_x) = (k_0 S/\pi)(-1)^m \sqrt{\varepsilon_m \varepsilon_{m'}}$$

$$\int_0^\infty \psi_{mn}(\tau\, a)\psi_{m'n'}(\tau\, a)\left[\frac{2}{\varepsilon_m}\delta_{mm'} + J_{m+m'}(2\tau\, l_y) \pm J_{m-m'}(2\tau\, l_y) \right] \frac{\tau\, d\tau}{\sqrt{k_0^2-\tau^2}}, \qquad (3.12)$$

which represents the normalized mutual radiation impedance of the modes (m,n) and (m',n') of the circular membrane vibrating and radiating into the two-wall corner subspace. The result in Eq. (3.12) is identical as that presented earlier in [3]. The limiting transitions considered can be interpreted physically in that way that the rigid baffle $x=0$ has been shifted infinitely from the membrane.

The obtained formulations of the radiation resistance are the basis for their low- and high-frequency approximations (cf. [5] and [6]).

3. THE MODAL RADIATION IMPEDANCE IN THE CASE OF THE LOW FREQUENCIES ($k_0 a << 1$)

The modal radiation resistance in the form of its single integral can be obtained from Eq. (3.10), if the integration is performed within the finite limits $(0 \le \tau \le k_0)$. In the case of the small values of the interference parameter $k_0 a << 1$, the following part of the corresponding integrand can be expressed as its expansion series taken around the point $\tau a = 0$

$$\Psi_{mn}(\tau)\Psi_{m'n'}(\tau) \cong \frac{(\tau a/2)^{m+m'}}{m!m'!\beta_{mn}\beta_{m'n'}}\left(1+\alpha_{mm'}(\tau a)^2 + O(\tau^4 a^4)\right), \tag{4.1}$$

where

$$\alpha_{mm'} = \frac{1}{\beta_{mn}^2} + \frac{1}{\beta_{m'n'}^2} - \frac{1}{4}\left(\frac{1}{m+1} + \frac{1}{m'+1}\right), \quad \alpha_m = \frac{1}{\beta_{mn}^2} + \frac{1}{\beta_{mn'}^2} - \frac{1}{4}\frac{1}{m+1}. \tag{4.2}$$

The integrand in Eq. (3.10) contains the double and triple Bessel function products. In the case of the double Bessel function products the second formula in Eq. (4.1) has been used whereas, in the case of the triple Bessel function products the first formula in Eq. (4.1) has been used. Introducing the two different formulations for the product in Eqs. (4.1) has been motivated by some further approximations of the modal radiation resistance and reactance.

The remaining part of the integrand in Eq. (3.10) containing the Bessel functions of the orders m+m’ and m=m’ has been left unchanged during the integration (cf. Eqs. (3.10) and (3.11)). Further, it is necessary to approximate the value of the following integral

$$I(k_0, c) = \int_0^{k_0} \tau^{v+2n+1} J_v(c\tau)\frac{d\tau}{\sqrt{k_0^2 - \tau^2}} \tag{4.3}$$

for $n = 0, 1, 2, ...$ and $v = m + m', \mathrm{m - m'}$ has been computed using the formula presented earlier in [7] as well as in Appendix A (Eq. (A.7)) giving its elementary form

$$I(k_0, \mathrm{c}) = n!(v+n)!k_0^{2n+v+1}\left(\frac{2}{k_0 c}\right)^n \sum_{s=0}^{n} \frac{(-1)^s (k_0 c/2)^s}{s!(v+s)!(n-s)!} j_{s+n+v}(k_0 c), \tag{4.4}$$

for $k_0 > 0$ *and* $v + \mathrm{n} + 1 > 0$. Further, using this formula results in the following elementary expression for the modal radiation resistance

$$\begin{aligned}
\theta_{mn,m'n'} \cong{}& \frac{\sqrt{\varepsilon_m \varepsilon_{m'}}(k_0 a)^2}{\beta_{mn}\beta_{m'n'}}\left(\frac{k_0 a}{2}\right)^{m+m'} \Big\{ q_1 \frac{\delta_{mm'} 2^{m+m'+1}}{\varepsilon_m (2m+1)!} \frac{1}{m!m'!}\Big[q_1 (-1)^{m'} i^{m+m'} j_{m+m'}(2k_0 l_y) \\
&+ (-1)^{m+m'} j_{m+m'}(2k_0 l_x) \pm (-1)^{m+m'} \cos(m+m')\beta j_{m+m'}(2k_0 l)\Big] \\
&+ \sum_{s=0}^{m'} \frac{(-1)^s}{s!(m-m'+s)!(m'-s)!}\Big[\pm q_1 (-1)^{m'} i^{m+m'} (k_0 l_y)^{s-m'} j_{s+m}(2k_0 l_y) \pm (-1)^m (k_0 l_x)^{s-m'} j_{s+m}(2k_0 l_x) \\
&+ (-1)^m \cos(m-m')\beta (k_0 l)^{s-m'} j_{s+m}(2k_0 l)\Big]\Big\},
\end{aligned} \tag{4.5}$$

for the interference parameter $k_0 a << 1$. The approximation in Eq. (4.5) has been derived neglecting the term $\alpha_{mm'}(\tau a)^2$ in the expansion series from Eq. (4.1). The factor $q_1 = 1$ given in Eq. (A.4) equal to the unity indicates only the terms in Eq. (4.5) in which $m + m'$ and $m - m'$ are even numbers.

In the specific case when $m = m' = 0$ in Eq. (4.5) the modal radiation resistance of the two axisymmetric modes $(0, n)$ and $(0, n')$ has been obtained in the form of

$$\theta_{0n,0n'} \cong \left[2\frac{(k_0 a)^2}{\beta_{0n}\beta_{0n'}} j_0(k_0 a) + \theta' \right] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \tag{4.6a}$$

where the second approximation term

$$\theta' \cong 2\frac{(k_0 a)^2}{\beta_{0n}\beta_{0n'}} \left[j_0(2k_0 l_x) + j_0(2k_0 l_y) + j_0(2k_0 l) \right] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \tag{4.6b}$$

represents the influence of the corresponding baffles of the three wall corner on the modal radiation resistance $\theta_{0n,0n'}$. The distance between the vibrating membrane central point and the central points of the three mirror images of the membrane is equal to $2l_x$, $2l_y$ and $2l$, respectively.

The modal radiation reactance χ corresponding to the modal radiation resistance θ has been obtained using the Hilbert transform [8]

$$\chi(k_0) = \frac{2k_0}{\pi} \int_0^\infty \frac{\theta(x)}{x^2 - k_0^2} dx, \tag{4.7}$$

the following formulation [7]

$$\int_0^\infty \frac{x^{3/2} J_{1/2}(bx) dx}{x^2 - k_0^2} = -\frac{\pi}{2} k_0^{1/2} N_{1/2}(bk_0), \tag{4.8}$$

as well as the relation $N_{1/2}(z) = -\sqrt{2/(\pi z)} \cos z$ where $N_{1/2}(z)$ is the Neumann function of the order ½, should also be used. The modal radiation impedance (resistance and reactance) has been expressed as the following single formulation

$$\xi_{0n,0n'} \cong 2\frac{(k_0 a)^2}{\beta_{0n}\beta_{0n'}} \left[h_0^{(1)}(k_0 a) + h_0^{(1)}(2k_0 l_x) + h_0^{(1)}(2k_0 l_y) + h_0^{(1)}(2k_0 l) \right] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \tag{4.9}$$

where $h_0^{(1)}(z) = j_0(z) + i n_0(z)$ is the spherical Hankel function of the first kind and $n_0(z)$ is the spherical Neumann function, both of the zero order. This spherical Hankel function has been expressed using the exponential function as $h_0^{(1)}(z) = \exp(iz)/iz$. It is worth noticing that using the two different approximate formulas from Eq. (4.1) has assured the absolute convergence of the integrals appearing during the application of the Hilbert transform while computing the radiation reactance directly from the corresponding radiation resistance.

4. FINAL REMARKS

The energy aspect of sound radiation of a vibrating circular membrane embedded into one of the three rigid baffles of the three wall corner situated perpendicularly to each other has been analyzed in this study. The main aim was to formulate the modal radiation impedance as a single integral in Eq. (3.10). This integral contains the triple product of the

Bessel functions of the integer order. The two different approximations of the double product of the Bessel function of the integer order in Eq. (4.1) have been used while evaluating the modal radiation resistance from Eq. (3.10), i.e. while computing the integral with the finite limits [0, k_0]. The approximation given in Eq. (4.5) containing the spherical Bessel functions can be useful for computations within the low frequency range, i.e. when the interference parameter $k_0 a << 1$. This approximation together with Eq. (4.6) makes it possible to analyze separately the main contribution of the influence of the two vertical baffles of the three wall corner subspace on the modal radiation resistance. The influence of the superposing acoustic waves reflected by the two vertical baffles on the modal radiation resistance has been described by the spherical Bessel functions with the corresponding arguments being the following values of the interference parameter: $2k_0 l_x$, $2k_0 l_y$ and $2k_0 l$. The modal radiation

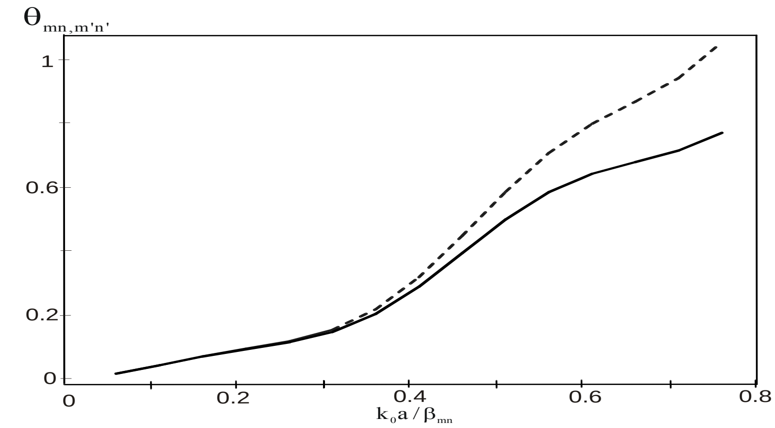

Fig.2 The sum of the modal radiation self-resistance of the mode (0,1) and the modal mutual resistance of the membrane mode (0,1) and its mirror image mode (0,1) as a function of the parameter $k_0 a/\beta_{01}$ for $l_y/a = 3$ and $\beta_{01} = 2.41$. The solid line– results from Eq. (3.12), the dashed line the results from the following expression

$$\theta_{01,01} \cong 2(k_0 a/\beta_{01})^2 \left[j_0(k_0 a) + j_0(2k_0 l_y) \right]$$

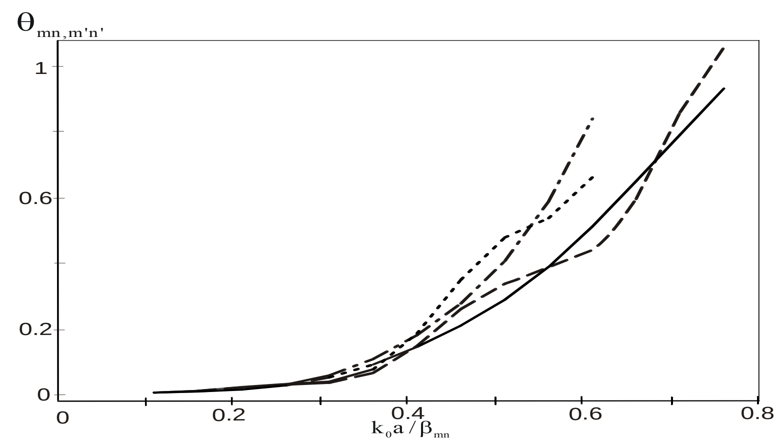

Fig.3 The sum of the modal radiation self-resistance $(1,1)$ and the modal mutual resistance of the membrane mode $(1,1)$ and its mirror image mode $(1,1)$ as a function of the parameter $k_0 a/\beta_{11}$ for $l_y/a = 3$ and $\beta_{11} = 3.83$. The solid and dashed lines result from Eq. (3.12). It has been assumed for the solid line that the vibrations of the membrane are described by the

function cosinus and for the dashed line by the function sinus. The remaining lines have been plotted from the following expression

$$\theta_{11,11} \cong \frac{(k_0 a)^4}{\beta_{11}^2}\left[\frac{j_0(k_0 a)}{k_0 a} \pm \frac{j_1(2k_0 l_y)}{2k_0 l_y} + j_2(2k_0 l_y)\begin{Bmatrix}1\\0\end{Bmatrix}\right],\ \begin{Bmatrix}1\\0\end{Bmatrix} \Rightarrow \begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}$$

For the dashed-dotted line it has been assumed that the vibrations are described by the cosinus function and for the dotted line by the sinus function

reactance $\chi_{mn,m'n'}$ for the mode pair (m, n) and (m', n') has been obtained from Eq. (3.10) by integrating within the infinite limits $(k_0 \le \tau < \infty)$. In the specific case when $k_0 a << 1$, the modal radiation impedance $\xi_{0n,0n'} = \theta_{0n,0n'} - i\chi_{0n,0n'}$ for the axisymmetric mode pair (0, n) and

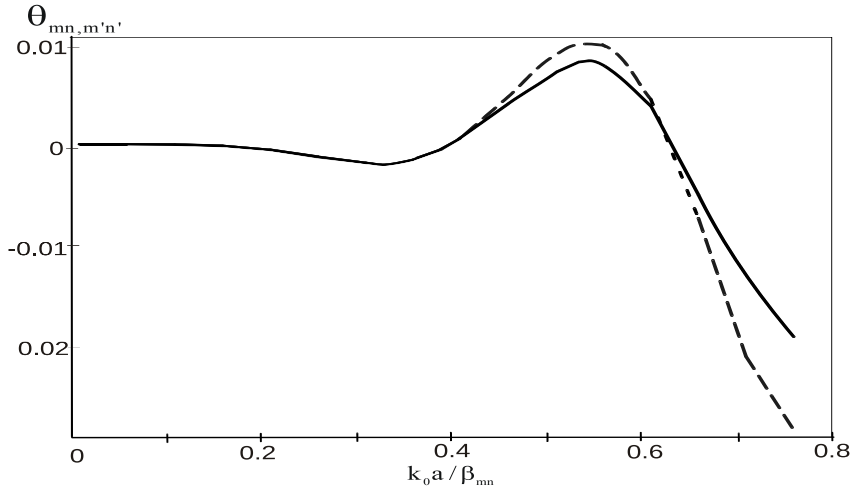

Fig.4 The modal mutual resistance of the membrane mode (2,1) and its mirror image mode (0,1) as a function of the parameter $k_0 a/\beta_{01}$ for $l_y/a = 3$, $\beta_{01} = 2.41$ and $\beta_{21} = 5.14$. The solid line– results from Eq. (3.12), the dashed line the results from the following expression

$$\theta_{21,01} \cong \frac{\sqrt{2}}{4}\frac{(k_0 a)^4}{\beta_{21}\beta_{01}} j_2(2k_0 l_y)\begin{Bmatrix}1\\0\end{Bmatrix}.$$

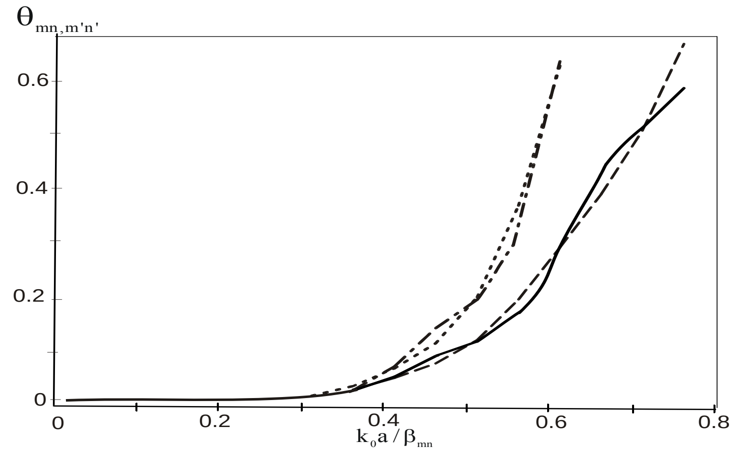

Fig.5 The sum of the modal radiation self-resistance $(2,1)$ and the modal mutual resistance of the membrane mode $(2,1)$ and its mirror image mode $(2,1)$ as a function of the parameter $k_0 a/\beta_{21}$ for $l_y/a = 3$ and $\beta_{21} = 5.14$. The solid and dashed lines result from

Eq. (3.12). It has been assumed for the solid line that the vibrations of the membrane are described by function cosinus and for the dashed line by function sinus. The remaining lines have been plotted from the following expression

$$\theta_{21,21} \cong \frac{(k_0a)^6}{4\beta_{21}^2}\left[\frac{j_2(k_0a)}{(k_0a)^2} \pm \frac{j_2(2k_0l_y)}{(2k_0l_y)^2} \mp \frac{j_3(2k_0l_y)}{2k_0l_y} + j_4(2k_0l_y)\begin{Bmatrix}1\\0\end{Bmatrix}\right], \begin{Bmatrix}1\\0\end{Bmatrix} \Rightarrow \begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix}$$

For the dashed-dotted line it has been assumed that the vibrations are described by the cosinus function and for the dotted line by the sinus function.

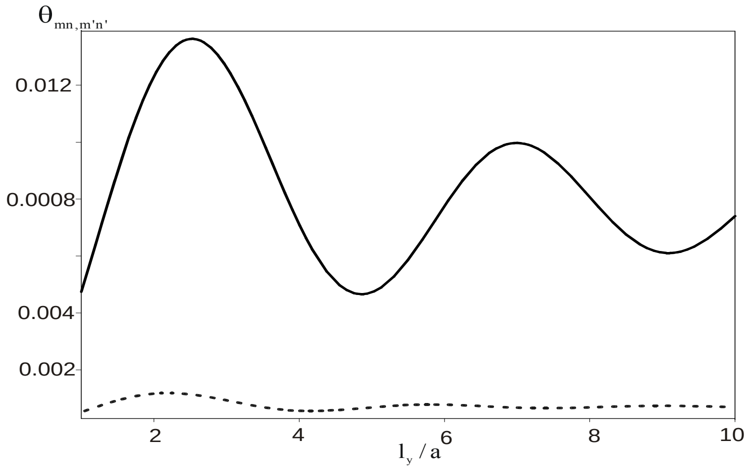

Fig.6 The modal radiation resistance as a function of the parameter l_y / a.
The solid line - the sum of the modal radiation self-resistance $(1,1)$ and the modal mutual resistance of the membrane mode $(1,1)$ and its mirror image mode $(1,1)$ for $k_0a/\beta_{11} = 0.2$.
The dashed line - the sum of the modal radiation self-resistance $(2,1)$ and the modal mutual resistance of the membrane mode $(2,1)$ and its mirror image mode $(2,1)$ for $k_0a/\beta_{21} = 0.2$.
The curves have been plotted by using the formulae from the legends of Fig. 3 and Fig.5 and it has been assumed that the vibration of the membrane is described by sinus function.

(0, n') has been approximated in Eq. (4.9) which contains the spherical Hankel functions of the first kind and the zero order, only.

Within the high frequency range $(k_0a > \beta_{mn})$, the modal radiation resistance $\theta_{mn,m'n'}$ can be approximated by exchanging the integral in Eq. (3.10) by the corresponding path integral, and further by computing the remainders at the poles of the integrand as well as by using the asymptotic method of computing the integrals. The modal radiation reactance $\chi_{mn,m'n'}$ can be approximated using the stationary phase method (cf. [5]).

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APPENDIX A

The values of the following integrals are necessary

$$I_1 = q_1 \int_0^{\pi/2} \begin{Bmatrix} \cos(m\alpha)\cos(m'\alpha) \\ \sin(m\alpha)\sin(m'\alpha) \end{Bmatrix} d\alpha = \frac{\pi}{4} q_1 \begin{Bmatrix} 2/\varepsilon_m \\ 1 \end{Bmatrix} \delta_{mm'}\,, \tag{A.1}$$

$$I_2 = q_1 \int_0^{\pi/2} \cos(2\tau l_x \cos\alpha)\cos(2\tau l_y \sin\alpha) \begin{Bmatrix} \cos(m\alpha)\cos(m'\alpha) \\ \sin(m\alpha)\sin(m'\alpha) \end{Bmatrix} d\alpha\,, \tag{A.2}$$

$$I_3 = q_2 \int_0^{\pi/2} \sin(2\tau l_x \cos\alpha)\cos(2\tau l_y \sin\alpha) \begin{Bmatrix} \cos(m\alpha)\cos(m'\alpha) \\ \sin(m\alpha)\sin(m'\alpha) \end{Bmatrix} d\alpha\,, \tag{A.3}$$

where

$m,m'=0,1,2,\ldots$, $l_x = l\cos\beta$, $l_y = l\sin\beta$ oraz $q_1 = \frac{1}{2}\left[1+(-1)^{m+m'}\right]$, $q_2 = \frac{1}{2}\left[1-(-1)^{m+m'}\right]$. (A.4)

The integrals I_2 and I_3 computed within the limits $(0, \pi/2)$ have been exchanged by the corresponding integrals computed within the limits $(0, \pi)$

$$\int_0^{\pi/2} g(\alpha)d\alpha = \frac{1}{2}\left[\int_0^{\pi/2} g(\alpha)d\alpha + \int_{\pi/2}^{\pi} g(\pi-\alpha)d\alpha\right],$$

giving

$$\int_0^{\pi/2} g(\alpha)d\alpha = \frac{1}{2}\int_0^{\pi} g(\alpha)d\alpha\,.$$

Applying the following Fourier series [8]

$$\cos(u\cos\varphi) = \frac{1}{2}\sum_{s=0}^{\infty}\varepsilon_s i^s[1+(-1)^s]J_s(u)\cos(s\varphi),$$

$$\sin(u\cos\varphi) = -\sum_{s=0}^{\infty} i^{s+1}[1-(-1)^s]J_s(u)\cos(s\varphi),$$

has made it possible to obtain the following values of the integrals

$$I_2 = q_1 I, \quad I_3 = -iq_2 I, \tag{A.5}$$

where it has been denoted

$$I = \frac{\pi}{4} i^{m+m'}\left\{(-1)^{m'}\cos(m-m')\beta J_{m-m'}(2\tau l) \pm \cos(m+m')\beta J_{m+m'}(2\tau l)\right\}, \tag{A.6}$$

and the plus sign in the symbol $\pm$ is related to the functions $\cos m\alpha$ and $\cos m'\alpha$ for m, m'=0,1,2,… whereas the minus sign is related to the functions $\sin m\alpha$ and $\sin m'\alpha$ for m, m'=1,2,…

While computing the integral in Eq. (4.3) the following more general formula has been used [7]

$$\int_0^a x^{\nu+2n+1}(a^2-x^2)^{\beta-1}J_\nu(cx)dx = \frac{1}{2}(\nu+1)_n\Gamma(\beta)a^{2n+2\beta+\nu}\left(\frac{2}{ac}\right)^{n+\beta}$$
$$\times\sum_{k=0}^{n}\frac{(-1)^k}{(\nu+1)_k}\binom{n}{k}\left(\frac{ac}{2}\right)^k J_{k+n+\beta+\nu}(ac), \tag{A.7}$$

where a>0, $\mathrm{Re}\beta > 0$, $\mathrm{Re}\nu > n-1$, $(\nu+1)_k = \frac{(\nu+k)!}{\nu!}$, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, and

$j_m(x) = \sqrt{\frac{\pi}{2x}}J_{m+1/2}(x)$ is the spherical Bessel function of the order m = 0, 1, 2,…

APPENDIX B

A detailed vibration analysis of the circular membrane has been presented in [3]. The equation of motion of the excited membrane including the acoustic attenuation in its amplitude form is

$$(\nabla^2 + k^2)W(r,\varphi) = -f(r,\varphi)/T - p(r,\varphi)/T, \tag{B.1}$$

where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}$, $\omega^2\sigma/T = k^2$, T is the stretching force, σ is the surface density of the membrane material, ω is the circular frequency of the excitation $f(r,\varphi)\exp(-i\omega t)$, $f(r,\varphi)$ is the excitation amplitude and $p(r,\varphi)$ is the amplitude of the acoustic pressure radiated by the membrane. It has been assumed that the radiation of the bottom side of the membrane (i.e. for z < 0) is suppressed.

The solution of Eq. (B.1) has been formulated as

$$W(r,\varphi)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}c_{mn}W_{mn}(r,\varphi), \tag{B.2}$$

and the corresponding eigenfunctions satisfying the equation $(k_{mn}^{-2}\nabla^2+1)W_{mn}(r,\varphi)=0$ of the mode (m,n) are

$$W_{mn}(r,\varphi)=\sqrt{\varepsilon_n}\,\frac{J_m(k_m r)}{J_{m+1}(\beta_{mn})}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}, \tag{B.3}$$

The eigenfunctions have been normalized by

$$\int_0^a\int_0^{2\pi}W_{mn}(r,\varphi)W_{m'n'}(r,\varphi)r\,dr\,d\varphi=\pi a^2\delta_{mm'}\delta_{nn'}, \tag{B.4}$$

where $\varepsilon_0=1$, $\varepsilon_m=2$ for $m\geq 1$, and $\beta_{mn}=k_{mn}a$ being the root of the frequency equation $J_m(\beta_{mn})=0$.

The equation of motion (B.1) has been rearranged and formulated as the following set of algebraic equations [3]

$$c_{mn}\left(\frac{k_{mn}^2}{k^2}-1\right)+i\varepsilon'\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}c_{m'n'}\zeta_{m'n',mn}=f_{mn}, \tag{B.5}$$

where $\varepsilon'=\rho_0 c/\sigma\omega$ determines the acoustic attenuation, and

$$f_{mn}=\frac{1}{S\sigma\omega^2}\int_0^a f(r,\varphi)W_{m'n'}(r,\varphi)r\,dr\,d\varphi. \tag{B.6}$$

Applying the set of algebraic equations (B.5) and the formulation of the modal radiation resistance $\xi_{mn.m'n'}$ leads to the following formulation for the total sound power radiated

$$\Pi=i\rho_0 c\frac{S\omega^2}{2\varepsilon'}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}c_{mn}^{*}\left[c_{mn}\left(\frac{k_{mn}^2}{k^2}-1\right)-f_{mn}\right]. \tag{B.7}$$