SUBDIFFRACTIVE PROPAGATION IN A BIDIMENSIONAL SONIC CRYSTAL

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Sonic crystals are media with a periodic modulation of the acoustic parameters, as the density and the bulk modulus. They have recently attracted a great interest, because of their potential applications in the control of sound propagation, used as reflectors, focusers or waveguides. All these properties are related with the dispersion introduced by the crystal anisotropy. We report on the nondiffractive propagation of sound in two-dimensional sonic crystals. It is shown that, for given frequencies and directions of incidence, a narrow sonic beam can propagate without diffractive broadening. Such nondiffractive sonic beams exist in crystals with perfect symmetry, and do not require the presence of defects, differently from other waveguiding phenomena reported previously. The cancellation of diffraction has been predicted using the plane-wave expansion method to evaluate the dispersion surfaces of the crystal and the spatial dispersion (isofrequency) curves. It occurs for frequencies and wavevectors for which dispersion curves have zero curvature, denoting a transition between focusing and defocusing regimes. By means of perturbative techniques, a simple analytical expression for the nondiffractive conditions has been obtained. The phenomenon is also demonstrated by numerical integration of the acoustic equations using the FDTD technique with very good agreement with the preliminary experimental results. *Support from Spanish MEC, project FIS2005-07931-C03-01, is acknowledged.*

INTRODUCTION

Sound crystals (SCs) are periodical structures of scatterers in an homogenoeus medium, i.e. are media with periodical modulation of their acosutical properties. The study of SCs was stimulated by the previous results in other field like optics [1], that permitted to use the so called photonic crystals to design materials with band gaps for the light propagation and to construct photonic crystal waveguides, and early similar results were obtained in the field od acoustics [2].

One of the most studied properties of the SCs is the existence of phohibited propagation acoustical frequency bands. These bands correspond to frequencies for which the acoustic wave does not propagate inside the crystal but it is reflected. Therefore the SCs are good candidates to manipulate band gaps and create waveguides and isolators.

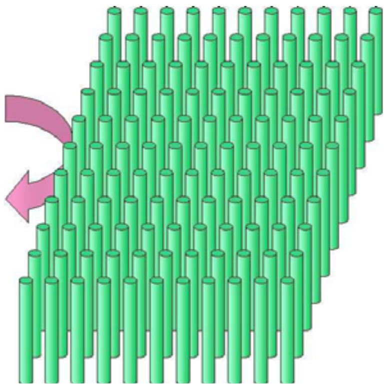


Fig.1 Bidimensional periodical matrix of cilindrical scatterers in a squared lattice can exhibit band gaps [2]

Most of the studies about SCs are devoted to the case of one dimension structures, what permits some analytical treatment, in oposition to the multidimensional cases which properties are numerically investigated either trhough the plane wave decompostion or finite difference methods. Also the major part of them concern only about the temporal dispersion properties introduced by the crystal.

In our case we will focus on a bidimensional crystal case. We will study no only the temporal dispersion variations introduced by the SC but also those on the spatial dispersion, i.e., how the periodicity of the crystal affects the diffraction properties. This idea has already led to the prediction of negative diffeation of light [3] and sound beams [4]. Recently it has been predicted for the photonic crystal case the existence of regions of total diffraction cancellation [5], permitting to obtain autocollimated or nondiffractive beams that could propagate long distances without spreading inside the crystal.

That is exactly the objective of this work. We study the nondiffrective propagation of acoustical waves inside sound crystals. We will use the plane wave decomposition method to find the nondiffractive regimes (Fig.2c) and will confirm the autocollimated propagation by the integration of the propagating equations with finite difference time-domain tecniques. We present an analitycal approach to tune the design of the experimental demonstration and show the first promising experimental results.

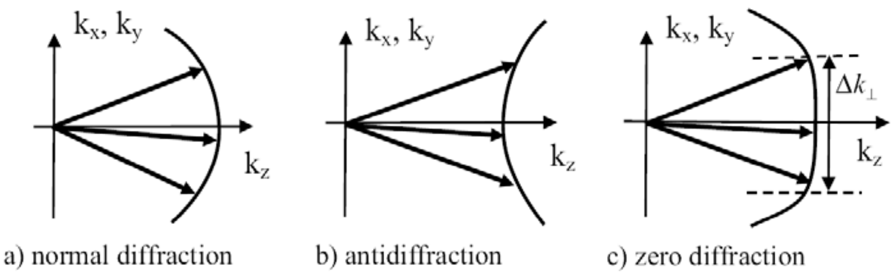


Fig.2 Geometrical interpretation of wave diffraction propagationg along z axis

1. THEORY

The acoustical propagation is governed by the set of equations

$$\rho\frac{\partial v}{\partial t} = -\nabla p, \qquad \frac{\partial p}{\partial t} = -B\nabla v, \tag{1}$$

where $\rho(r)$ and $B(r)$ are the medium density and bulk modulus (space dependent), $p(r,t)$ is the scalar pressure and $v(r,t)$ is the vector velocity. Assuming an harmonic dependence for the fields, the equation for a wave of given ω frequency is given by the eigenvalues equation

$$\frac{\omega^2}{\bar{B}(r)}p(r) + \nabla\left(\frac{1}{\bar{\rho}(r)}\nabla p(r)\right) = 0, \tag{2}$$

where the upper bar means that the quantity is normalised to that of the homogenous host medium. Being the lattice vectors $R = \{R = n_1 a + n_2 a;\ n_1, n_2 \in N\}$, with a the scatterers distance and the reciprocal lattice vectors $G = \{G : G\cdot R = 2\pi n;\ n \in N\}$, and developing the variables in that last vector base, $\bar{\rho}(r)^{-1} = \sum_G \rho_G^{-1} e^{i\mathbf{Gr}}$, $\bar{B}(r)^{-1} = \sum_G b_G^{-1} e^{i\mathbf{Gr}}$ and $p(r) = e^{ikr}\sum_G p_{k,G}\, e^{i\mathbf{Gr}}$ (Bloch-Floquet theorem) we obtain the eigenvalues equation

$$\sum_{\mathbf{G}'}\left[\omega^2 b^{-1}_{\mathbf{G}-\mathbf{G}'} - \rho^{-1}_{\mathbf{G}-\mathbf{G}'}(\mathbf{k}+\mathbf{G})\cdot(\mathbf{k}+\mathbf{G}')\right] p_{\mathbf{G}'} = 0. \tag{3}$$

To solve this equation will permit to know the crystal band structure, and therefore the possible existence of forbidden bandgaps together with the isofrequency contours that will allow to notice the nondiffractive zones existence.

2. NUMERICAL SIMULATIONS

In our case the resolution of equation (3) was done taking into account 361 plane waves, in order to assure convergence. We show the results for the prohibited bands corresponding to the first and second bands (Fig.3).

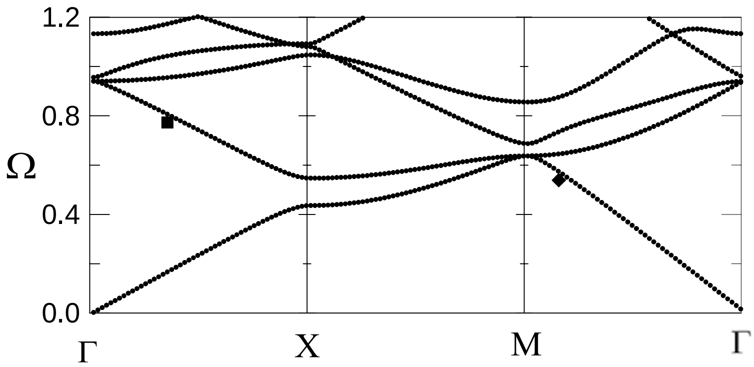


Fig.3 Band structure for a squared lattice of cylinders of radius 1mm immersed in water, separated a=5,25mm. Black squares mark the nondiffractting points

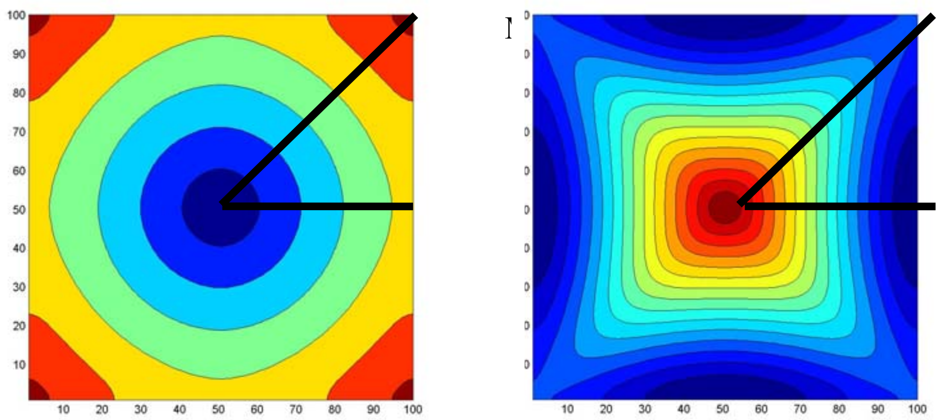


Fig.4 Isofrequency lines for the first (a) and second (b) bands. The crystal is formed by steel cylinders of radius 1mm, spaced 5.25 mm and inmersed in water

As can be observed in Fig.4, the isofrequency curves change from concave to convex. In the surfaces corresponding to that transition we found locally plane segments, directly related to the frequencies and directions where the beam spreading is compensated by the crystal anisoptropy and the subdiffractive propagation occurs.

In order to validate the occurrence of nondiffractive propagation at these values we have integrated numerically the equations (1), using the finite differences time domain method. The incident beam on a simulates a plane transducer with diameter of 3 cm emitting with variable frequency ω. We plot both results for the first and second band in Fig.5.

3. ANALITYCAL RESULTS

The analysis of the field structure using the plane wave decomposition implies to consider an infinite number of modes in the equation (3), but for the practical numerical solution we take into account a finite but still large number of modes. Nevertheless, if we set the system conditions close to the nondiffractive propagation zone, we can develope an analytic theory considering only the three most relevant modes, the homogeneous mode and the closest lower order modes (with wave vectors $\boldsymbol{k}$, $\boldsymbol{k}+\boldsymbol{G}_1$ and $\boldsymbol{k}+\boldsymbol{G}_2$, see Fig.6).

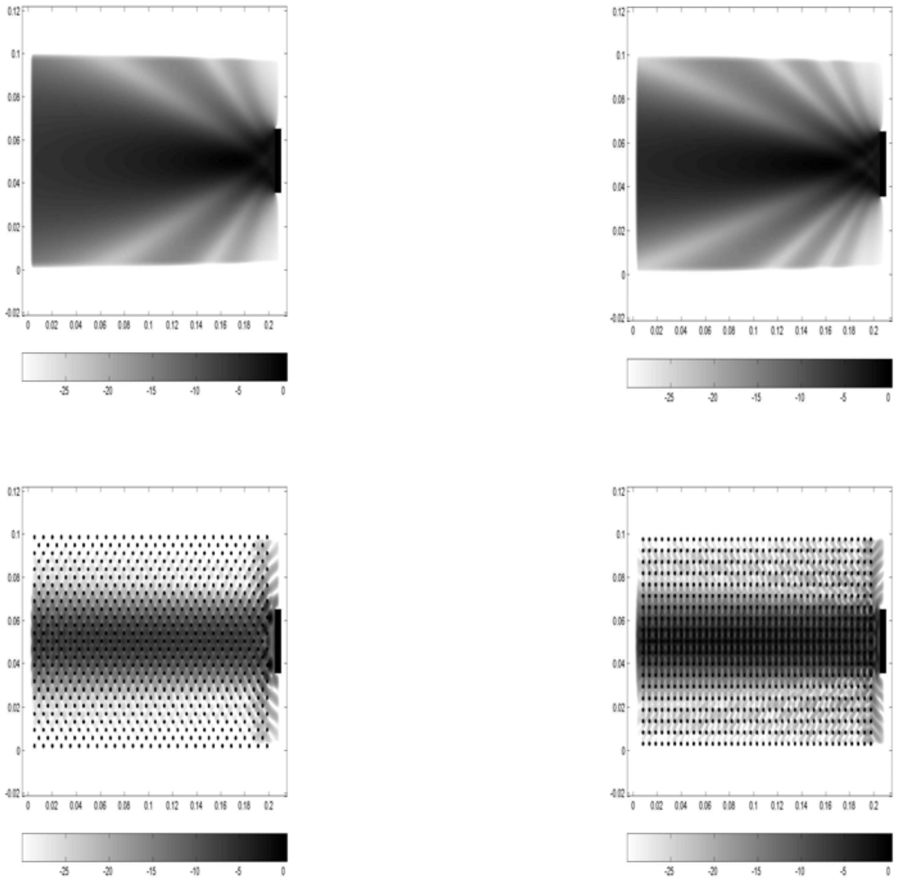


Fig.5 Numerical simulation of the nondiffractive propagation for the first band (Left), f=154 kHz in the [1,1] crystal direction, and for the second band (Right), f=217 kHz in the [1,0] crystal direction. The upper plots depict the free propagation for both frequencies for the sake of comparison

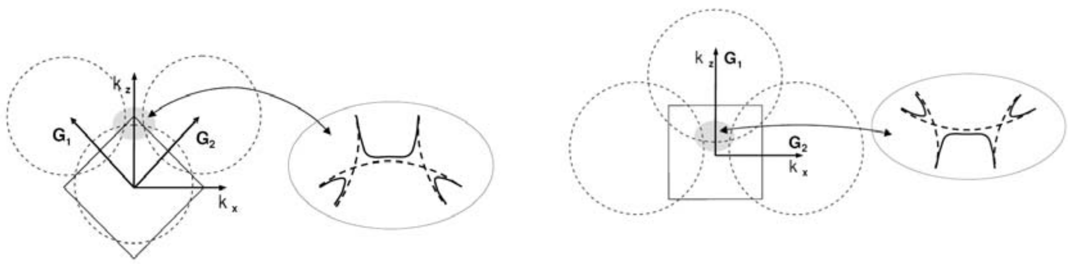


Fig.6 Schematical draw showing the non diffractive zone (shadowed) as the interaction between only three modes; Left) first band, right) second band

Assuming a small filling factor (cylinder radius<<distance between them), strong acoustic impedance differences between host and scatterers and a frequency close to the forbbiden gap, we arrive to the following analytical expressions that provide the frequency and wave number deviations $\delta\Omega_{ND}^{(1)}$ and $\delta K_{ND}^{(1)}$ of the nondiffractive regime from the frequency and wave number of the band gap:

$$\delta\Omega_{ND}^{(1)} = f^{2/3} + O\left(f^{4/3}\right), \tag{5}$$

$$\delta K_{ND}^{(1)} = f^{2/3} - f^{4/3} + \frac{3}{4} f^{2} + O(f^{7/3}), \tag{6}$$

and give the minimum beam size d that can propagate without diffraction, being $d \approx 2\pi / \delta k_{ND} \approx 2\pi\, f^{-2/3}$. Figure 7 shows the great agreement between this analytical approach with three modes and the numerical simulation with the complete 361 modes set.

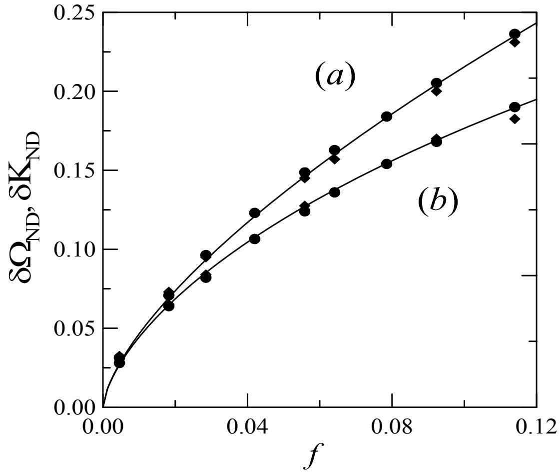
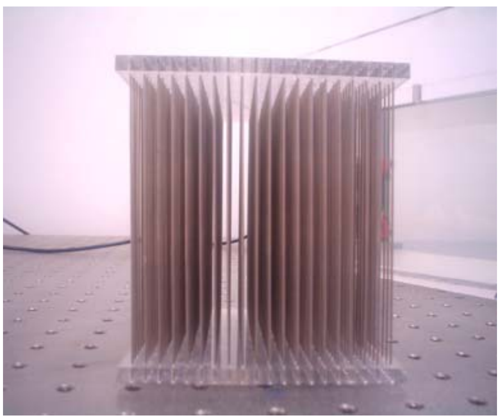


Fig.7 Dependence of frequency (a) and wave number (b) deviations from the bandgap for nondiffractive propagation. Symbols correspond to numerical calculations and lines to the analytical results.

4. EXPERIMENT

The experiments to achieve and characterize the nondiffractive propagation in a bidimensional array are still in progress but the first results are very promising and agree spectacularly with the theoretical and numerical calculations. The experiment setup in a plane transducer placed in front of a sonic crystal made of a 10x10 squared array of steel rods of radius 0.8 mm spaced 5.25 mm, and we compare the radiation at the exit of the crystal with the free propagation case. Figure 8 shows the crystal and the basic measuring scheme with a needle hydrophone.



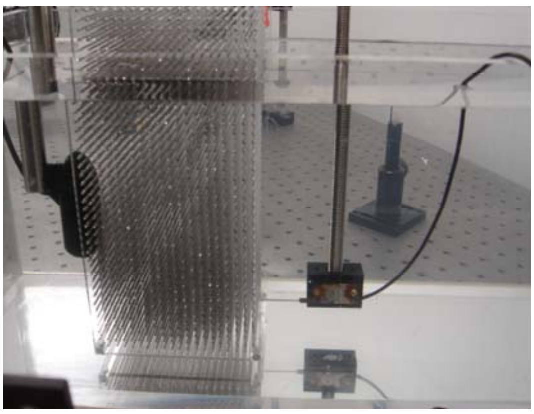


Fig.8 Left: the squared array; the active crystal direction is perpendicular to the rods. Right: Measuring scheme

According to the theory, propagation is nondiffractive for 165 kHz, with beam directed along (1,1) direction (first band) and for 232 kHz, with beam directed along (1,0) direction (second band). The Figure 9 show the measurements at 232 kHz compared to the numerical simulations with an extraodinary agreement. The small deviations of the free propagation case from the ideal transverse profile are due to interferences with the bottom and surface in the small tank. The measured effect has been plotted in Figure 10 for f = 240kHz, comparing input and output with the free propagation case.

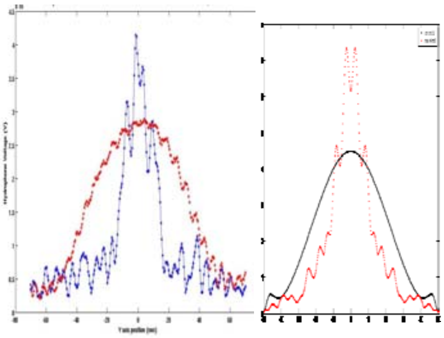


Fig.9 Left: experimental result of nondiffractive propagation (blue) vs. free propagation (red) at the second band. Right: numerical simulation with finite differences time domain tecniques for the same case

In order to visualize the importance of the effect, we have plotted plotted in Figure 10 the transversal mapping of the beam for f = 240kHz, comparing the input (distance from transducer z=1 mm) and the output from the crystal with the free propagation case (z=110 mm)

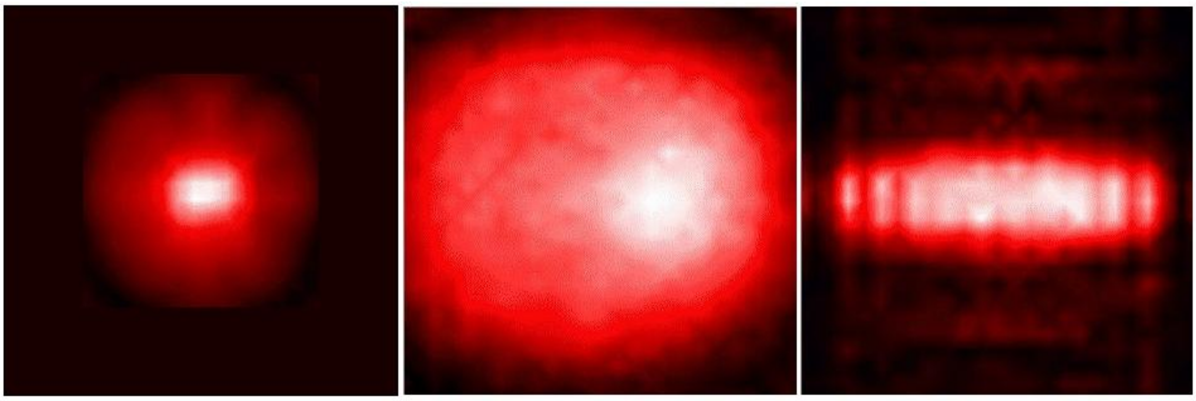


Fig.10 Left: experimental measurement of beam input at 240kHz (60 mm x 60mm area). Center: experimental measurement of beam free diffraction at z=110 mm. Right: experimental measurement of crystal effect at the same distance z=110 mm

5. CONCLUSIONS

We have demonstrated theoretically and experimentally the nondiffractive propagation of sound in two-dimensional sonic crystals. We have derived a simplified analytical approach in order to design our experiments which show very nice agreement with the numerical calculations.

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