A NOVEL METHOD FOR AMPLITUDE AND PHASE MEASUREMENTS BASED ON MULTIPLE SAMPLES

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Precise amplitude and phase measurement of a narrowband signal is required in many practical applications related to underwater sensing, navigation, and positioning. In this paper, we propose a novel method for digital signal acquisition based on multiple signal sampling. This method is an extension of quadrature sampling where amplitude and phase of a harmonic signal can be obtained based on only two samples taken within one period. The advantage of our generalization are reduced errors caused by quantization noise of A/D converters.

INTRODUCTION

Amplitude and phase of a harmonic signal often convey important information. For instance, several underwater navigation systems rely on relative phase measurements for bearing and azimuth estimation as well as for range determination [1, 2]. Robust and precise methods are required to assure high system dynamics and robustness in the presence of noise. Several algorithms for amplitude and phase measurements, both analog and digital, have been proposed in the literature [1, 3, 4]. In this paper, we present a novel digital method for amplitude and phase measurement of harmonic signals that offers better performance in the presence of quantization noise.

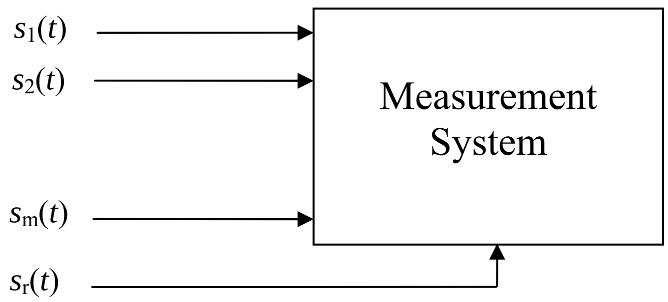


Fig.1 Measurement System

In general, several harmonic signals at the same frequency f are applied to the measurement system as shown in Fig. 1. The function of the system is to recover amplitudes and phases relative to a reference signal sr(t) from its discrete samples. We will discuss here a single signal s(t) given by equation (1) since generalization to several signals is straightforward:

$$s(t) = A\cos(\omega t + \varphi). \tag{1}$$

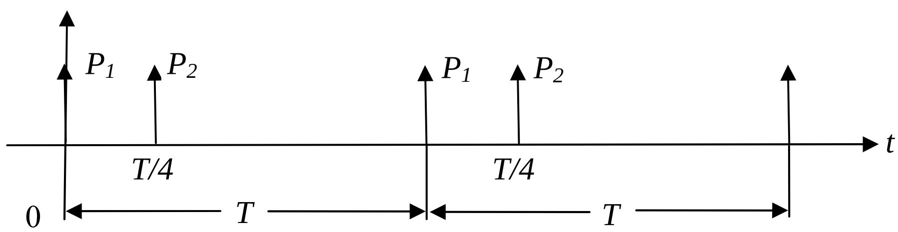
Equation (2) represents the reference signal

$$s_r(t) = A_r\cos(\omega t) \tag{2}$$

where $\omega = 2\pi f = 2\pi/T$.

The object of the measurement is to estimate the amplitude A and the phase φ of s(t) based on discrete samples of s(t).

A classical digital algorithm for amplitude and phase estimation is based on so-called quadrature sampling. Assuming that amplitude and phase of the signal s(t) are constant within one period T, amplitude A and the phase φ can be derived from only two samples P1 and P2 (called a doublet) taken in quadrature, that is, T/4 apart as shown in Fig. 2.



1.

Fig.2 Quadrature Sampling

From equations (1) and (2) we can obtain the following discrete samples:

$$P_1 = s(0) = A\cos(\varphi) \quad (3)$$

$$P_2 = s(T/4) = A\cos((\omega T/4) + \varphi) = A\cos(\varphi + \omega T/4) = A\cos(\varphi + \pi/2) = -A\sin(\varphi) \quad (4)$$

From equations (3) and (4), we can obtain the amplitude and phase of $s(t)$, that is,

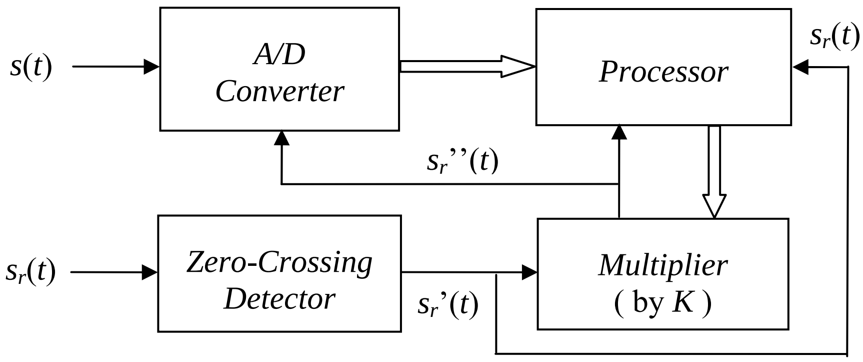
$$A = \sqrt{(P_1)^2 + (P_2)^2} \quad \text{and} \quad (5)$$

$$\varphi = \tan^{-1}(-P_2/P_1)\,. \quad (6)$$

The minimum doublet sampling interval is every period T; therefore, the maximum doublet sampling frequency is fd =1/T. The actual sampling frequency is determined by Nyquist rates as given by bandwidths of A(t) and φ(t).

1. PROBLEM FORMULATION

Fig.3 shows the general block diagram of the proposed measurement system for a single signal s(t). A harmonic reference signal sr(t) is converted to square waveform sr'(t) by a zero-crossing detector. The positive slopes of this waveform provide period timing to the processor. The frequency multiplier generates the sr''(t) waveform at Kf frequency that provides sampling instances to the processor. The parameter N is controlled by the processor based on external conditions. The A/D converter digitizes s(t) at suitable sampling instances and transfers the data to the processor. During A/D conversion a quantization errors are introduced to each sample.

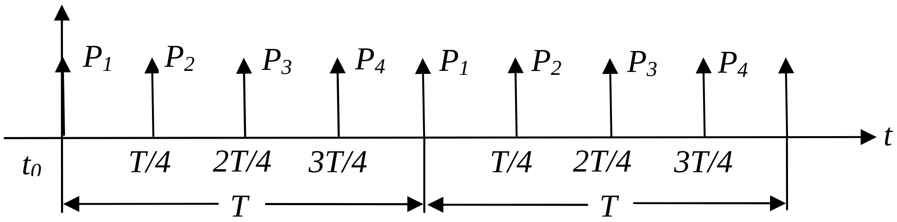


2.

Fig.3 Block Diagram of Measurement System

2. QUADRUPLET SAMPLING

We will generalize quadrature sampling here by taking four samples (a quadruplet) per period T, instead of two samples (doublet) as in the classic approach. Period T is divided into four quadrants (T/4) and one sample is taken at the beginning of each quadrant as illustrated in Fig.4. This results in four samples, P1, P2, P3, and P4 per period T. This situation corresponds to N = 1 in Fig.3.



3.

Fig.4 Quadruplet Sampling

According to equations (1) and (2), we can obtain the following:

$$P_1 = A\cos(\varphi) \quad (7)$$

$$P_2 = A\cos(\omega(T/4)+\varphi) = A\cos(\varphi+\pi/2) = -A\sin(\varphi) \quad (8)$$

$$P_3 = A\cos(\omega(2T/4)+\varphi) = A\cos(\varphi+\pi) = -A\cos(\varphi) \quad (9)$$

$$P_4 = A\cos(\omega(3T/4)+\varphi) = A\cos(\varphi+3\pi/2) = A\sin(\varphi). \quad (10)$$

From equations (7) through (10), we obtain:

$$P_1 - P_3 = 2A\cos(\varphi) \quad (11)$$

$$P_4 - P_2 = 2A\sin(\varphi) \quad (12)$$

and, therefore:

$$A = \sqrt{(P_4-P_2)^2+(P_1-P_3)^2}\Big/2 \quad (13)$$

$$\varphi = \tan^{-1}((P_4-P_2)/(P_1-P_3)). \quad (14)$$

The advantage of this approach is an expected better system performance in the presence of sampling errors since the amplitude and phase calculations are based on four samples (quadruplet), rather than the two samples (doublet) in the classic approach. Here, again, the frequency of quadruplet sampling is determined by the bandwidths of A(t) and φ(t).

3. GENERALIZED SAMPLING (K-PLET)

The approach can be generalized further by dividing each quadrant T/4 into N equal parts. The first sample is taken at the beginning of each part. This leads to uniform sampling frequency fs = 4N/T within one period or K= 4N samples (K-plets) per period where $N \geq 1$ is an arbitrary integer. This generalization is illustrated in Fig.5, where Pij denotes a sample in the ith quadrant (i=1, 2, 3, 4) and j denotes the sample number within the ith-quadrant (j=1, 2, 3, 4, …, N).

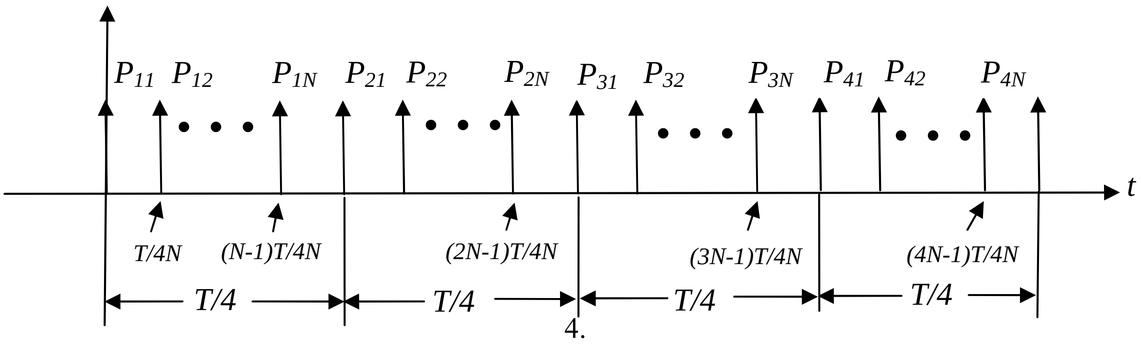


Fig.5 Generalized Sampling (4N-plets)

According to equations (1) and (2), we can obtain the following:

$$P_{11} = A\cos(\omega(1-1)T/4N+\varphi) = A\cos(\varphi) \quad (15)$$

$$P_{1j} = A\cos(\omega(j-1)T/4N+\varphi) = A\cos((j-1)\pi/2N+\varphi)$$
$$= A\cos((j-1)\pi/2N)\cos(\varphi) - A\sin((j-1)\pi/2N)\sin(\varphi) \quad (16)$$

$$P_{21} = A\cos(\omega((1-1)T/4N)+\varphi+\pi/2) = A\cos(\varphi+\pi/2) = -A\sin(\varphi) \quad (17)$$

$$P_{2j} = A\cos(\omega(j-1)T/4N+\varphi+\pi/2) = A\cos((j-1)\pi/2N+\varphi+\pi/2)$$
$$= -A\sin((j-1)\pi/2N+\varphi)$$
$$= -A\sin((j-1)\pi/2N)\cos(\varphi) - A\cos((j-1)\pi/2N)\sin(\varphi) \quad (18)$$

$$P_{31} = A\cos(\omega((1-1)T/4N)+\varphi+\pi) = A\cos(\varphi+\pi) = -A\cos(\varphi) \quad (19)$$

$$P_{3j} = A\cos(\omega(j-1)T/4N+\varphi+\pi) = A\cos((j-1)\pi/2N+\varphi+\pi)$$
$$= -A\cos((j-1)\pi/2N+\varphi)$$
$$= -A\cos((j-1)\pi/2N)\cos(\varphi) + A\sin((j-1)\pi/2N)\sin(\varphi) \quad (20)$$

$$P_{41} = A\cos(\omega((1-1)T/4N)+\varphi+3\pi/2) = A\cos(\varphi+3\pi/2) = A\sin(\varphi) \quad (21)$$

$$P_{4j} = A\cos(\omega(j-1)T/4N+\varphi+3\pi/2) = A\cos((j-1)\pi/2N+\varphi+3\pi/2)$$
$$= A\sin((j-1)\pi/2N+\varphi)$$

$$= A\sin((j-1)\pi/2N)\cos(\varphi) + A\cos((j-1)\pi/2N)\sin(\varphi). \quad (22)$$

From equations (15) through (22), we obtain:

$$P_{21} - P_{41} = -2A\sin\varphi \quad (23)$$

$$\sum_{j=1}^{N}(P_{1j} - P_{3j}) = 2A(\sum_{j=1}^{N}\cos((j-1)\pi/2N))\cos\varphi - 2A(\sum_{j=1}^{N}\sin((j-1)\pi/2N))\sin\varphi \quad (24)$$

$$P_{11} - P_{31} = 2A\cos\varphi \quad (25)$$

$$\sum_{j=1}^{N}(P_{4j} - P_{2j}) = 2A(\sum_{j=1}^{N}\sin((j-1)\pi/2N))\cos\varphi + 2A(\sum_{j=1}^{N}\cos((j-1)\pi/2N))\sin\varphi. \quad (26)$$

From equations (23) and (24), equation (27) follows, and from equations (25) and (26), equation (28) follows:

$$\sum_{j=1}^{N}\left[(P_{1j} - P_{3j}) - (P_{21} - P_{41})\sin((j-1)\pi/2N)\right] = 2A\cos\varphi(\sum_{j=1}^{N}\cos((j-1)\pi/2N)) \quad (27)$$

$$\sum_{j=1}^{N}\left[(P_{4j} - P_{2j}) - (P_{11} - P_{31})\sin((j-1)\pi/2N)\right] = 2A\sin\varphi(\sum_{j=1}^{N}\cos((j-1)\pi/2N)) \quad (28)$$

Let:

$$X_1 = \sum_{j=1}^{N}\left[(P_{1j} - P_{3j}) - (P_{21} - P_{41})\sin((j-1)\pi/2N)\right]$$

$$X_2 = \sum_{j=1}^{N}\left[(P_{4j} - P_{2j}) - (P_{11} - P_{31})\sin((j-1)\pi/2N)\right] \text{ and}$$

$$X_3 = \sum_{j=1}^{N}\cos((j-1)\pi/2N),$$

then equations (27) and (28) can be rewritten as:

$$X_1 = 2A(\cos\varphi)X_3 \quad (29)$$

$$X_2 = 2A(\sin\varphi)X_3. \quad (30)$$

It can be proven that $X_3 > 0$ and, with equations (29) and (30), one can finally obtain equations (31) and (32), that is:

$$A = \sqrt{X_1^2 + X_2^2}\Big/(2X_3) \quad (31)$$

$$\varphi = \tan^{-1}(X_2/X_1). \quad (32)$$

Equations (31) and (32) reduce to (13) and (14) for $N = 1$, as expected.

4. SIMULATION RESULTS AND CONCLUSIONS

The expected advantage of the presented method is its robustness in the presence of quantization noise related to the resolution of A/D converters. Because of the complicated nature of the general case of K samples per period, we resort to computer simulation to gain some indication of improvements.

For simulation, the following parameters were assumed for the signal: A = 1, φ = 45o, f = 10kHz. Each sample is contaminated by random, additive, independent, quantization noise ε (error) with uniform distribution of $-4/256 \leq \varepsilon \leq 4/256$. Each simulation was performed 1000 times for different values of K.

Table 1 shows the results. The average estimated amplitude $\overline{A}$ and phase $\overline{\varphi}$ were calculated together with their standard deviations σA and σϕ. The improvement in term of smaller standard deviations in comparison to classic quadrature sampling (with K=2) is shown for both amplitude and phase estimates. As we can see, the improvement is substantial for K = 4, but is less pronounced for larger K. This fact must be considered when making compromise between accuracy, sampling rate and A/D converter resolution. More investigations are needed to fully explore those aspects.

Tab.1 Simulation Results

K	$\overline{A}$	σ_A	Improvement	$\overline{\varphi}$	σ_ϕ	Improvement
2	0.99981	0.0086849	N/A	44.9939	0.52516	N/A
4	1.00030	0.0064099	26.20%	44.9991	0.36781	30.00%
8	0.99997	0.0059593	31.40%	45.0096	0.34504	34.30%
12	0.99957	0.0060931	29.90%	45.0117	0.33568	36.10%
16	1.00000	0.0059374	31.60%	44.9906	0.35030	33.30%
20	0.99994	0.0061364	29.40%	44.9913	0.34834	33.70%
24	1.00020	0.0062669	27.80%	45.0156	0.34400	34.50%
28	0.99999	0.0062921	27.60%	45.0116	0.35748	31.90%

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